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Specialized Content Knowledge for Teaching Equations: A Case Study of Secondary Mathematics Teachers

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The Mathematical Knowledge for Teaching (MKT) framework conceptualizes specialized content knowledge (SCK) through the dimensions of justification, explanation, and representation. Drawing on this framework, this study investigates secondary school mathematics teachers' specialized content knowledge in the teaching of first-order equations with one unknown and identifies the instructional responsibilities associated with this topic. Designed as a qualitative case study, the research was conducted with three secondary school mathematics teachers. Data were collected through an equation knowledge test for teaching, semi-structured interviews, and classroom observations conducted during the instruction of first-order equations and were analyzed using analytical rubrics aligned with the three dimensions of SCK. The findings indicate that although teachers demonstrated procedural fluency in solving equations, their justifications and explanations frequently lacked conceptual depth. Teachers encountered difficulties when responding to students' questions about the relational meaning of the equality sign, the interpretation of variables, and the equivalence of different solution strategies. With respect to representation, teachers relied predominantly on symbolic forms, with limited integration of verbal explanations and visual models to support conceptual understanding. The study specifies topic-specific instructional responsibilities embedded in specialized content knowledge for first-order equations and offers a structured analytical perspective to inform future research and the design of teacher education programs in algebra instruction.

Introduction

Teachers' mathematical knowledge plays a central role in shaping the quality of mathematics instruction and students' opportunities to develop conceptual understanding. Early work on teacher knowledge emphasized that effective teaching requires more than general subject matter knowledge; teachers also need knowledge that enables them to represent, explain, and transform content in ways that are meaningful for learners (Shulman, 1986). Building on this perspective, Ball and colleagues developed the Mathematical

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Knowledge for Teaching (MKT) framework, which conceptualizes the knowledge required for mathematics teaching under two broad domains: subject matter knowledge and pedagogical content knowledge (Ball et al., 2008; Hill et al., 2008). Within this framework, Specialized Content Knowledge (SCK) refers to mathematical knowledge that is specific to the work of teaching and is frequently used in classroom instruction.

SCK is particularly important because mathematics teachers are expected not only to solve problems correctly but also to explain why mathematical procedures work, evaluate the validity of students' alternative strategies, and make connections among concepts, procedures, and representations. In classroom practice, this knowledge becomes visible when teachers answer students' "why" questions, justify mathematical claims, select appropriate representations, and interpret students' reasoning (Ball et al., 2008). Therefore, examining teachers' SCK provides a useful perspective for understanding how mathematical knowledge is transformed into instructional action.

This perspective is especially relevant for algebra instruction. Algebra is a fundamental component of mathematical literacy because it supports students' ability to generalize relationships, reason with symbols, and solve problems involving unknown quantities (Kieran, 2007; Öztürk & Kaplan, 2019). Within algebra, first-order equations with one unknown are particularly important because they constitute one of students' first systematic encounters with symbolic reasoning and form a basis for later algebraic learning. However, research shows that students often experience difficulties in understanding equations, especially in relation to the meaning of equality, variables, unknowns, and equation-solving procedures (Dede & Argün, 2003; Kieran, 2007). These difficulties indicate that equation teaching requires more than presenting rules and solution steps; it requires teachers to make the underlying mathematical structure explicit.

Studies on algebra teaching show that teachers' content knowledge significantly affects the quality of instructional practices and student learning (Hill et al., 2008; König et al., 2021). Teachers who view algebra mainly as a set of rules may emphasize procedural operations while providing limited conceptual explanations of equality, variables, and solution methods (Welder & Simonsen, 2011). Such a procedural orientation may limit students' opportunities to develop a relational understanding of equations. For this reason, examining teachers' SCK in equation instruction is important for understanding how teachers explain mathematical ideas, evaluate students' reasoning, and use representations to support conceptual learning.

Although many studies have examined teachers' SCK through tests or open-ended questions, fewer studies have investigated how SCK is enacted in actual classroom practices. Yet knowing mathematics for teaching and using that knowledge in instruction are not identical (Ball et al., 2001). Lin, Chin, and Chiu (2011) synthesized the mathematics-related duties described in the MKT framework and conceptualized SCK through three interrelated dimensions: explanation, justification, and representation. These dimensions provide an analytical basis for examining how teachers use mathematical knowledge while explaining procedures, justifying students' solution strategies, and selecting or connecting representations during instruction.

In this context, the aim of the present study is to examine secondary school mathematics teachers' specialized content knowledge regarding first-order equations with one unknown and to identify the subject-specific SCK duties and responsibilities that emerge during equation teaching. To achieve this aim, teachers' SCK was investigated through both an

Equation Knowledge Test for Teaching (EKTT) and classroom observations. Specifically, the study focuses on the explanation, justification, and representation dimensions of SCK in order to provide a more practice-oriented understanding of teachers' mathematical knowledge in the teaching of first-order equations with one unknown.

Theoretical Framework and Related Literature

Specialized Content Knowledge

Specialized Content Knowledge (SCK) is one of the components of subject matter knowledge within the Mathematical Knowledge for Teaching (MKT) framework developed by Ball and colleagues (2008). The MKT framework was proposed to distinguish the mathematical knowledge required for teaching from the mathematical knowledge used in other professions. According to this framework, effective mathematics teaching requires not only knowledge of mathematics itself but also knowledge that enables teachers to use mathematics in ways that support students' understanding. Within the subject matter knowledge domain, SCK occupies a distinctive position because it represents mathematical knowledge that is unique to the work of teaching.

Unlike common content knowledge, which involves correctly performing mathematical procedures or solving problems, SCK focuses on understanding and communicating the underlying mathematical ideas. For example, while many individuals may correctly solve an equation, mathematics teachers are additionally expected to explain why a solution method works, evaluate alternative student strategies, identify misconceptions, interpret students' reasoning, and connect different mathematical representations. Thus, SCK extends beyond procedural competence and involves using mathematical knowledge to make concepts, relationships, and structures accessible to learners (Ball et al., 2008; Hill et al., 2008). Because these forms of knowledge are central to classroom instruction, SCK constitutes the theoretical focus of the present study.

During the development of the MKT framework, Ball et al. (2008) identified a variety of mathematics-related tasks that teachers regularly perform in classroom settings. These tasks include explaining mathematical ideas, responding to students' questions, evaluating solution strategies, selecting appropriate representations, making connections among mathematical concepts, and examining mathematical structures. According to Ball et al. (2008), effective mathematics teaching depends not merely on possessing mathematical knowledge but on mobilizing that knowledge while carrying out these instructional responsibilities.

Building on this perspective, Lin, Chin, and Chiu (2011) synthesized these mathematics-related teaching tasks and conceptualized SCK through three interrelated dimensions: explanation, justification, and representation. Rather than focusing on isolated teaching actions, this framework provides a systematic way of examining how teachers employ mathematical knowledge during instruction.

The explanation dimension refers to teachers' ability to explain mathematical concepts, procedures, rules, and algorithms by emphasizing the underlying mathematical ideas. Such explanations help students develop conceptual understanding rather than relying solely on procedural knowledge (Richland et al., 2012). The justification dimension concerns evaluating the validity of mathematical claims, arguments, and solution strategies, as well as providing mathematical reasons for why a particular solution or approach is correct or incorrect (Ball et al., 2005). The representation dimension involves selecting, using, and connecting verbal, symbolic, numerical, and visual representations to support mathematical understanding. Effective use of multiple representations has been shown to facilitate conceptual learning and strengthen students' problem-solving performance (Moseley & Brenner, 1997).

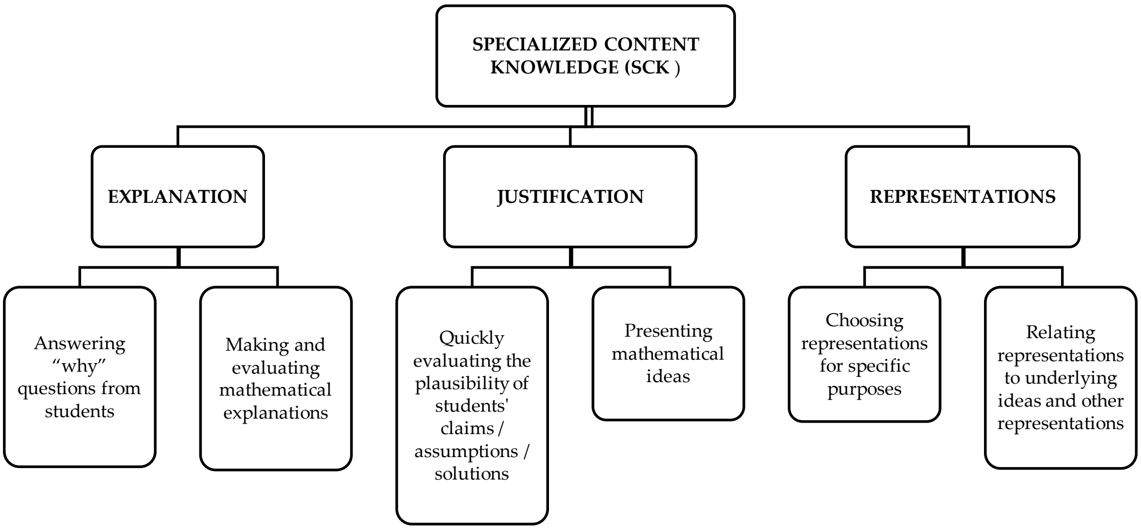


Figure 1. Subdimensions of Specialized Content Knowledge (adapted from Lin et al., 2011)

These three dimensions provide a coherent analytical framework for examining how teachers enact specialized content knowledge in classroom settings. Because the purpose of the present study is to investigate secondary school mathematics teachers' specialized content knowledge regarding first-order equations with one unknown, the explanation, justification, and representation dimensions proposed by Lin et al. (2011) were adopted as the theoretical framework guiding data collection, analysis, and interpretation. Both the Equation Knowledge Test for Teaching (EKTT) and the classroom observations were designed and analyzed with reference to these three dimensions.

Algebra Instruction and Teacher Knowledge

Algebra, and particularly equation solving, constitutes a cornerstone of mathematical literacy and plays a critical role in students' academic development (Kieran, 2007; Sert Çelik & Masal, 2018). First-order equations with one unknown represent students' initial formal encounter with algebraic reasoning and serve as the foundation for more advanced algebraic work (Kieran, 2007; Knuth et al., 2008). Research consistently shows that students develop persistent misconceptions related to equality, unknowns, and algebraic operations (Dede & Argün, 2003; Kieran, 2007). Among the most prevalent are the operational interpretation of the equal sign, the conflation of variable and unknown, and the overgeneralization of arithmetic procedures to algebraic contexts.

Teachers' content knowledge in this domain is a key determinant of instructional quality and student learning outcomes (Hill et al., 2008; Ball et al., 2008; König et al., 2021). Research suggests that middle school mathematics teachers often demonstrate relatively strong awareness of students' misconceptions; however, they experience substantial difficulties in transforming this awareness into effective instructional strategies supported by deep subject matter knowledge (Cenikli, Şahin, & İnan Tutkun, 2026). Asquith, Stephens, Knuth, and Alibali (2007) found that many teachers hold a procedural rather than relational understanding of the equality sign, which limits their ability to address students' conceptual confusion. Similarly, Huang and Kulm (2012) documented gaps in prospective middle-grade teachers' algebraic knowledge, particularly in areas requiring explanatory and justificatory reasoning.

Recent reviews of MKT-based research further point to a growing need for topic-specific and practice-oriented analyses that connect teachers' disciplinary knowledge with classroom enactment, particularly in algebra instruction (Grigaliūnienė et al., 2025). Despite the growing body of research grounded in the MKT framework, existing studies have predominantly examined teachers' specialized content knowledge (SCK) through written assessments or decontextualized tasks. While such approaches provide valuable information about teachers' knowledge profiles, they offer limited insight into how SCK is enacted during actual classroom instruction, particularly within specific mathematical topics (Ball et al., 2008). Moreover, empirical studies that integrate teachers' test-based SCK profiles with systematic analyses of their instructional practices remain scarce, especially at the middle school level and within the domain of equation solving (Grigaliūnienė et al., 2025). Given that equations constitute a conceptually demanding area of algebra instruction, this lack of practice-oriented, topic-specific research represents a significant gap in the literature.

Addressing this gap is essential for developing a nuanced understanding of how teachers' specialized content knowledge functions in real teaching contexts and for informing mathematics teacher education grounded in classroom realities. Accordingly, this study adopts the MKT framework to examine secondary school mathematics teachers' SCK related to first-order equations with one unknown at the 7th-grade level. Teachers' SCK was investigated through both an Equation Knowledge Test for Teaching (EKTT) and systematic classroom observations, with particular attention to the subdimensions of explanation, justification, and representation. Through this integrated approach, the study aims to identify subject-specific SCK behaviors enacted during equation teaching and to contribute to a more practice-oriented understanding of teacher knowledge in mathematics education (Ball et al., 2008; Lin et al., 2011).

Purpose and Importance of Research

The aim of the study is to examine the specialized content knowledge of secondary school mathematics teachers about 7th-grade first-degree equations with one unknown using the MKT theoretical framework and to determine the duties and responsibilities of specialized content knowledge that teachers are expected to have in the teaching processes related to this topic. Teachers' content knowledge in algebra has been shown to significantly influence instructional quality and student learning outcomes (Dede & Argün, 2003; Kutluca & Birgin, 2007). Accordingly, investigating teachers' subject-specific knowledge as it is enacted in classroom practice is essential for understanding the relationship between mathematical knowledge and teaching (Ball et al., 2008; Hill et al., 2008).

The focus on first-degree equations is grounded in three considerations. First, algebra, and particularly equations, constitutes a foundational component of mathematics learning and serves as a building block for advanced mathematical concepts (Kutluca & Birgin, 2007; Sert Çelik & Masal, 2018). Second, SCK represents a dimension of knowledge that is specific to the work of teaching and becomes visible through instructional actions (Ball et al., 2008). Third, contemporary curriculum reforms emphasize meaningful learning and conceptual understanding, which require teachers to employ representations, provide conceptually grounded explanations, and evaluate students' alternative strategies in real time (Ball et al., 2005; Ministry of National Education, 2025). By examining teachers' SCK both through a structured test and through systematic observation of classroom practices, this study aims to provide a topic-specific and practice-oriented analysis of how specialized content knowledge is enacted in equation teaching and to contribute to the field of mathematics teacher education.

This study examines secondary school mathematics teachers' specialized content knowledge regarding equations with one unknown within the framework of Mathematical Knowledge for Teaching (MKT). In line with this purpose, the research questions of the study are as follows:

(1) How is the specialized content knowledge of secondary school mathematics teachers manifested across the subdimensions of justification, explanation, and representation?
(2) What subject-specific instructional duties and responsibilities, aligned with the subdimensions of justification, explanation, and representation, are observable in the classroom practices of secondary school mathematics teachers during the teaching of first-order equations with one unknown?

Method

Research Design

This study employed a qualitative case study design to examine secondary school mathematics teachers' specialized content knowledge (SCK) regarding first-order equations with one unknown. The case study approach was selected because the central purpose of the inquiry is to gain an in-depth understanding of how specific teachers enact SCK within their own classroom contexts, rather than to generate findings that generalize across a broader population. Case study research is well suited to investigations that require attending to contextual particularities, examining multiple data sources within a bounded setting, and analyzing complex instructional phenomena in their natural environment (Creswell & Creswell, 2018; Yin, 2018). Given the practice-oriented nature of the research questions and the need to capture enacted knowledge rather than abstract competency, the case study design offered the most appropriate methodological alignment. In this study, teachers' SCK was analyzed through two complementary data collection instruments: an open-ended test structured in a semi-interview format and systematic classroom observations.

Working Group

The study group consisted of three secondary school mathematics teachers working in a public school in İzmir who taught 7th-grade mathematics. Participants were selected through convenience sampling, defined as selecting individuals who are easily accessible and willing to participate (Golzar, Noor, & Tajik, 2022). An additional criterion was that teachers had at least five years of experience teaching 7th grade, where first-order equations with one unknown are introduced. Given the in-depth nature of the analysis, a small sample of three teachers was considered sufficient. Classroom instruction was documented through video and audio recordings. Video recording enabled the analysis of teachers' use of written representations on the board and their spatial positioning during instruction, while audio recording provided a clear and unambiguous transcript of verbal exchanges between teachers and students. Using both modalities together ensured that neither verbal nor visual instructional information was lost during transcription and analysis. The secondary school mathematics teachers who participated in the research graduated from education faculties' four-year primary mathematics teaching programs, completing coursework in both mathematics and mathematics education. The teachers' names were kept confidential, and codes such as T1, T2, and T3 were used in place of their names. Demographic information is summarized in Table 1.

Table 1. Demographic information of the secondary school mathematics teachers

Teacher Code	Gender	Education Status	Teaching Experience	Equations Teaching Experience
T1	Male	Bachelor's Degree	17 years	13 years
T2	Female	Bachelor's Degree	12 years	11 years
T3	Male	Bachelor's Degree	13 years	7 years

Data Collection Tools

The study used three data collection instruments: the Equation Knowledge Test for Teaching (EKTT), the Lesson Observation Form (LOF), and a Lesson Interview Form. The Lesson Interview Form was used in post-lesson discussions with teachers to clarify and contextualize episodes observed during classroom instruction and thus functioned as a follow-up instrument rather than a primary, independent data source. Together, these three instruments provided complementary perspectives on teachers' SCK: the EKTT assessed their written, reflective knowledge; the LOF captured enacted knowledge during instruction; and the follow-up interviews allowed for elaboration of ambiguous or notable classroom events.

Equation Knowledge Test for Teaching (EKTT)

The EKTT was prepared in accordance with a semi-structured interview format consisting of open-ended questions to examine teachers' SCK regarding the subject of first-order equations with one unknown. This research focused on achievements related to the Equality and Equation sub-learning area of the 7th-grade Algebra Teaching Area. The learning outcomes examined are given in Table 2.

Table 2. Learning outcomes examined

Understands the principle of conservation of equality.
Recognizes first-order equations with one unknown and establishes the equation appropriate to given real-life situations.
Solves first-order equations with one unknown.
Solves problems that require establishing a first-order equation with one unknown.

Ministry of National Education.

The EKTT was developed based on the specialized content knowledge (SCK) framework of the MKT model. Following Ball et al. (2005), who conceptualized SCK in terms of explanation, justification, and representation, the test was designed to assess teachers' conceptual understanding and its use in instructional contexts (Aslan-Tutak & Köklü, 2016). A draft version consisting of 12 open-ended questions was prepared, ensuring balanced coverage of the four learning objectives related to first-order equations with one unknown. The draft test was piloted with five secondary school mathematics teachers in İzmir, and revisions were made based on their responses and feedback. The final EKTT included 12 questions distributed equally across the three SCK components (Items 1–4: justification, Items 5–8: explanation, and Items 9–12: representation). Representative questions from each subdimension are presented in the Findings section alongside teachers' responses to illustrate both the instrument's design and the criteria applied in analysis. The complete EKTT, including all 12 open-ended items, is provided in Appendix A.

Lesson Observation Form (LOF)

The Lesson Observation Form (LOF) was developed to examine secondary school mathematics teachers' instructional practices in terms of SCK-related duties. Observed situations were discussed with teachers in post-lesson interviews. The LOF included six duties aligned with the three SCK components: (1) responding to students' "why" questions, (2) making and evaluating mathematical explanations, (3) presenting mathematical ideas, (4) evaluating the plausibility of students' claims and solution strategies, (5) selecting representations for specific purposes, and (6) relating representations to underlying ideas and to other representations. These six duties were selected because they correspond directly to the three SCK subdimensions articulated by Lin et al. (2011), with two duties per subdimension, thereby allowing a balanced, subdimension-level analysis of classroom enactment. Each duty was derived from Ball et al.'s (2008) original enumeration of teaching-specific mathematical tasks, adapted to the instructional context of first-order equation teaching.

The LOF was designed to align with the EKTT so that findings from both instruments could be compared. Content validity was established through expert review by two mathematics education researchers, and revisions were made accordingly. Video recordings and observation notes were analyzed based on the SCK components and duties specified in the LOF.

Data Collection Process

Prior to data collection, participants were informed about the purpose and procedures of the study. Teachers first completed the EKTT, and their open-ended responses were evaluated using a prepared rubric. Responses requiring clarification were addressed in follow-up interviews. Classroom observations were conducted throughout the unit on first-order equations with one unknown to examine how teachers enacted their SCK during instruction. Lessons were video recorded with permission, enabling detailed analysis of instructional events. A total of 30 lesson hours (10 per teacher) were recorded, excluding sessions focused solely on individual student problem-solving. The researcher attended the lessons to take observation notes using the LOF alongside video recordings. All recordings were transcribed verbatim. To enhance credibility, two field experts reviewed the video data, and transcription procedures were checked to ensure consistency.

Analysis of Data

Qualitative data were analyzed using content analysis, descriptive analysis, and constant comparison techniques. The data sources included the EKTT, classroom observations, and interviews, and findings from each source were compared to reach comprehensive conclusions. Analytical rubrics developed by the researchers were used to evaluate both EKTT responses and observation data. Content analysis enabled the organization of teachers' responses and instructional practices under themes derived from the theoretical framework (Yıldırım & Şimşek, 2013). Coding was conducted within a general framework (Strauss & Corbin, 1990), whereby initial codes were established based on the SCK framework and expanded through data-driven additions. Teachers' responses and observed practices were categorized into four performance levels: none (Level 0), inappropriate (Level 1), appropriate but incomplete (Level 2), and appropriate and complete

(Level 3). The units of analysis were defined based on recurring expressions and behaviors. Representative examples from EKTT responses and classroom observations were incorporated into the rubric.

The rubrics were developed in three stages. First, the researchers derived an initial set of criteria from the SCK framework and the relevant literature on equation teaching (Ball et al., 2008; Lin et al., 2011). Second, these criteria were refined through an iterative examination of pilot EKTT responses, during which the researchers identified recurring patterns of conceptual completeness and incompleteness that corresponded to the four performance levels. Third, the rubrics were reviewed and validated by two external mathematics education researchers before being applied to the main dataset. The rubrics served as both a classification tool and a basis for comparative analysis: once responses and classroom behaviors were categorized, the distribution of levels across subdimensions enabled systematic identification of SCK strengths and gaps. Separate rubrics were constructed for EKTT responses and observation data, since the cognitive demands of written explanations differ from those of in-the-moment instructional decisions. Crucially, these rubrics were designed for classification and interpretive analysis rather than for scoring or quantitative aggregation. An example of the rubric used for analyzing the explanation component of the EKTT is presented in Table 3, and the corresponding observation rubric is presented in Table 4.

Table 3. Analytical rubric used to score the Equation Knowledge Test for Teaching (explanation dimension)

Dimension	Level 0 (None)	Level 1 (Inappropriate)	Level 2 (Appropriate, Incomplete)	Level 3 (Appropriate, Complete)
Explanation	No answer. Left blank.	An action is left incomplete after writing a number or drawing a shape. There is an inappropriate mathematical operation, model, or problem. Explaining the situation with another figure but not clarifying the desired point.	Explaining the situation appropriately but with incomplete statements. The desired situation is not expressed clearly. For example: explaining that the balance is not disturbed when –13 is moved in 107=4x−13 using the reverse-transaction method, but without clearly stating why −13 must be moved.	Explaining the situation completely and clearly. For example: showing with the balance model that adding +13 to both sides leaves 4x alone; connecting this to the balancing both sides method; explaining the inverse-operation shortcut as a condensed form of that method.

Table 4. Analytical rubric used to score observation findings (explanation dimension)

Dimension	Level 0 (None)	Level 1 (Inappropriate)	Level 2 (Appropriate, Incomplete)	Level 3 (Appropriate, Complete)
Explanation	No explanation was given.	The explanations provided are merely descriptions of the process	The desired situation is explained appropriately but with incomplete expressions. For example, providing explanations about the reasons for each	The desired situation is explained completely and clearly. For example, explaining each solution step, introducing the balancing both sides

Dimension	Level 0 (None)	Level 1 (Inappropriate)	Level 2 (Appropriate, Incomplete)	Level 3 (Appropriate, Complete)
		steps, or an inappropriate model is used. For example, attempting to explain an equation with a negative-integer solution using the balance model.	solution step but limiting those explanations to the inverse operation method without first introducing the balancing both sides method.	method before using the inverse method as a shortcut, and connecting both to the balance model.

Each teacher's observation transcripts were reviewed twice as a quality-control measure to verify transcription accuracy and to ensure that no substantive classroom exchange had been omitted or misrepresented. The two reviews were conducted independently before any analytical coding began. The coding procedure involved five passes through each teacher's 10-hour dataset. The first two passes were used to establish an initial coding scheme grounded in the SCK subdimensions. The third and fourth passes allowed the researchers to compare codes across the dataset and refine category boundaries. The fifth pass served as a final verification to confirm coding consistency and to identify any episodes that required discussion between the two analysts. This iterative process was adopted to maximize analytical depth and to reduce the risk of premature or unstable coding.

Validity and Reliability

To enhance internal validity, participants were informed about the purpose and procedures of the study to reduce potential reactivity and encourage natural classroom practices. The EKTT was developed based on relevant literature, and all interview and observation data were transcribed and checked to prevent data loss. Consistency between themes and rubric dimensions was ensured through continuous comparison of data sources. Data triangulation was achieved by using multiple instruments, including the EKTT, observations, and interviews.

External validity was supported by providing detailed descriptions of the research design, study group, data collection procedures, analysis processes, and the MKT framework used in the study. Reliability was addressed through both internal and external procedures. Two researchers independently analyzed the data, and coding consistency was examined by comparing agreements and disagreements. Interrater reliability was calculated using the Miles and Huberman (1994) formula, yielding an agreement rate above 80%, which is considered acceptable. External reliability was strengthened by clearly documenting data collection and analysis procedures to allow replication in similar contexts.

Findings

The research findings consist of two parts: the findings obtained from the Equation Knowledge Test for Teaching (EKTT) and the findings obtained from observation data.

Findings from the Equation Knowledge Test for Teaching (EKTT)

Teachers' responses to the EKTT were evaluated using analytical rubrics across the three SCK subdimensions: justification, explanation, and representation. Performance levels were categorized as none (0), inappropriate (1), appropriate but incomplete (2), and appropriate and complete (3). The rubric levels described here serve as an analytical classification tool to characterize qualitative patterns in teachers' SCK, not to produce a numerical performance score. The research questions concern how SCK manifests and which instructional duties are enacted, not the ranking of individual teachers.

Table 5. Summary of teachers' performance levels across SCK components

SCK Component	General Pattern Observed	Dominant Level
Justification	Teachers correctly identified appropriate strategies but provided limited conceptual reasoning.	Appropriate, Incomplete
Explanation	Explanations were often procedurally accurate but lacked deep conceptual justification.	Appropriate, Incomplete
Representation	Teachers were generally more successful in constructing symbolic expressions; however, conceptual alignment with problem contexts was sometimes weak.	Mixed (Complete / Incomplete)

Across all three components, teachers were generally able to produce procedurally correct responses. However, their explanations and justifications frequently lacked conceptual depth. In the justification dimension, teachers recognized correct strategies but often failed to articulate the underlying mathematical structure. In the explanation dimension, rule-based reasoning was dominant, and teachers tended to rely on general equality principles without addressing equation-specific constraints (e.g., restrictions related to division by algebraic expressions). Performance was relatively stronger in the representation dimension, particularly in forming symbolic expressions. Nevertheless, some responses revealed a tendency to focus on obtaining numerically correct results rather than aligning representations with the student's reasoning presented in the problem context. Overall, the findings suggest that teachers' algebra knowledge is largely procedural, while the conceptual and evaluative dimensions of SCK, particularly those requiring deeper justification and context-sensitive reasoning, remain underdeveloped.

One of the justification questions relates to the learning outcome, "To recognize first-degree equations with one unknown and to formulate a first-degree equation with one unknown that is appropriate for given real-life situations."

> *"During a lesson on solving first-degree equations with one unknown, Teacher 1 (T1) asks his students to solve the equation 34+5n = 9+6n. While looking at the students' methods, he noticed that two students were discussing the solution of the equation. Meanwhile, one of the students asked her teacher, "We both have the same answers, but my friend solved it differently. Wouldn't it be in the way I did?"*

First Student's Solution	*Second Student's Solution*
$34+5n=9+6n$	$34+5n=9+6n$
$25+5n=6n$	$34+5n-(9+6n)=0$
$25=n$	$25-n=0$
	$n=25$

Indicate whether the second student's equation-solving strategy is always usable. (Tick the appropriate option—true, false, not sure—and write your reason.)

The teachers' answers to the first question of the justification dimension described above are shown in Table 6.

Table 6. Answers to the first question of the justification dimension

Teachers	Answers Given
Teacher 1	The difference of two equal expressions is equal to 0. Therefore, it can be used at any time.
Teacher 2	In first-order equations with one unknown, moving one side of the equation to the opposite side and resetting the other side and finding the root of the equation is a strategy that always gives the correct answer.
Teacher 3	The aim here is to find the unknown. To find the unknown, X must be left alone. When the known and the unknown are pulled to separate sides, X will be left alone. The value of the remaining X is the sought-after answer. Strategy can be used.

When the teachers' responses were evaluated, it was evident that their answers required revision. Although all three stated that the strategy could always be used, their justifications only partially addressed why it was valid. T1 made a general statement- "The difference between two equal expressions equals 0"-but did not relate this reasoning explicitly to the given equation. T2 accurately described the student's procedural steps yet failed to explain why those steps preserved equality. T3 referred to the inverse operation method commonly used in solving equations; however, this explanation did not fully correspond to the student's actual solution process, since the student did not initially separate knowns and unknowns but applied this reasoning only at the final stage. What was expected was an explicit explanation that the balancing both sides method was employed: subtracting from both sides' yields , followed using the inverse operation in the final step.

One of the explanatory questions was related to the learning objective "Solving first-degree equations with a single unknown." The question and teacher responses are given below.

The students of Teacher 2 were trying to solve the following equation.

4. (x-3) =2. (x-3)

The first student suggested dividing both sides by 2 to obtain:

2. (x-3) = x-3

Then, he suggested dividing both sides by x-3, but the second student replied, "You cannot divide both sides by x-3." In response, a third student asked, "If you can divide both sides by 2, why cannot you divide by x-3?

What kind of explanation do you think Teacher 2 should make regarding the situation defended by the first student?

Teachers' answers to the question of the explanation dimension described above are given in the table below.

Table 7. Responses to the third question of the explanation dimension

Teachers	Answers Given
Teacher 1	It is not mathematically wrong to divide both sides by x-3. It can be done. However, when x-3 is divided, the definition of the equation disappears as the unknown disappears.
Teacher 2	While solving an equation, adding, and subtracting the same number to both sides of the equation does not change the equality, and multiplying and dividing both sides of the equation by the same number does not break the equality.
Teacher 3	If we divide both sides by x-3, we get 2=1. This is against equality. In other words, the unknown variable has disappeared from the equation. However, we are looking for the unknown.

When the responses in Table 7 are evaluated, it is seen that the explanations of T1 and T3 are appropriate but not sufficient. T2's explanation is not appropriate for the given situation. When the explanations of T1 and T3 are examined, they are aware that dividing both sides of the equation by x-3 is not mathematically wrong, but doing such an operation specifically for the given equation will break the equality. However, neither teacher mentioned that the value 3, which makes the denominator 0 when the equation is divided by x-3, cannot be included in the solution set. When three is written instead of x, the equation is satisfied since 0=0. Therefore, dividing by x-3 would not be appropriate. What was expected from teachers was to include this situation in their statements. For this reason, their answers were not evaluated under the appropriate, complete category. Although the explanation made by T2 is mathematically correct, it is erroneous when explicitly evaluating the problem in question. T2 mentioned that the equality situation will not be broken by dividing both sides of the equation by the same number. However, in the case of this problem, dividing both sides by x-3 will not ensure the conservation of equality. The dialog between T2 and the researcher regarding this situation is given below.

R: *Can you look at question 7 again?*

T2: *Okay. Yes, I could not write much. It can be divided by x-3.*

R: *What kind of result do we obtain when we divide by x-3?*

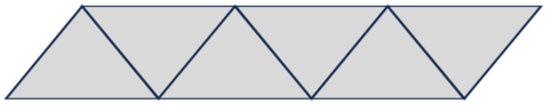
T2: *If we send x-3 from both sides, 2 = 1. In this case, there is no equation. We tell students that dividing both sides by the same number does not break equality, but they should be told that they should try and see if it breaks.*

When the dialog above is examined, it is understood that T2 considers dividing both sides of the equation by the same number and dividing by an expression containing an unknown in the same situations. However, he overlooks that the value of an expression containing an unknown, which makes the denominator zero, should be evaluated on an equation-specific basis. The teacher's extreme adherence to the rule prevents him from thoroughly evaluating the question.

The answers given by teachers to the second question regarding the representation dimension are given below. The second of the representation questions is related to the learning outcome, "Recognizes first-order equations with one unknown and constructs a first-order equation with one unknown appropriate to given real-life situations."

A student creates geometric shapes with matchsticks in her hand. He states that the perimeter of the figure below, consisting of 6 triangles, can be calculated as follows:

"I use three matchsticks for each triangle. I count the right edge of each triangle twice, except for the last triangle. So, I have to take these off."



What algebraic representation best represents the student's triangle description, with "n" representing the total number of toothpicks used? Explain, please.

Teachers' answers to the second question of the representation dimension are given in the table below.

Table 8. Answers to the second question of the representation dimension

Teacher Codes	Answers Given
Teacher 1	Judging from the figure, there are 13 toothpicks and 6 triangles. In other words, an expression should be written such as n = 13 t = 6. If 3 matchsticks are used to make 1 triangle, 3x6 matchsticks are used to make 6 triangles. We represent this with 3.t. Except for the last triangle I made, there are 5 triangles left. There are 5 matchsticks counted twice in these triangles. If we subtract these, we get an expression like 3t-(t-1) = n. If we write 6 instead of t and 13 instead of n, we see that the equation is satisfied.
Teacher 2	n = 3t-3. From the total number of toothpicks, we subtract 3, which is the number of matches in the last triangle. (Because he said except)
Teacher 3	If there are 13 matchsticks, n=13, t=6 number of triangles If 3 matchsticks are used for each triangle, we can write 3.t. It is satisfied if the equation is written as 3t-t+1=13.

When the teacher responses given in Table 8 are evaluated, it is seen that T1 can write the mathematical expression appropriate to the situation given in the question. T2 gave an incorrect answer. He misinterpreted the phrase "except for the last triangle I made" in the question and subtracted the number of matches in the last triangle from the total number of toothpicks. Therefore, the answer given needs to be revised. Although T3 wrote a mathematical expression, he did not explain the justification of the expression he wrote. During the interview, he was asked to explain the question. The dialog between T3 and the researcher is given below.

A: *I want you to review question 10 again.*

T3*: Yes. This question was one that I had a tough time with.*

A: *Okay. You wrote a statement. Can you explain how you wrote this?*

T3: *Let me try to remember again. Because I wrote it without being too sure. One minute. Ok. I can tell. I wrote it on paper. I wrote 3t because it said three matchsticks for a triangle. This was a total matchstick.*

A: *Then, you subtracted t. What does this mean?*

T3: *I removed this because it said to exclude the last triangle. However, I could not remember why I removed 1. Let me think for a while. Oh, okay. Now, we started with n=13 and t=6. When you write these in the equation, it should satisfy them. That is why I added one last. It provided the equation.*

When the dialog above is examined, T3 has subtracted the variable *t*, which is used to represent the rank of the last triangle, from the total number of toothpicks he wrote.

Therefore, he made a mistake in the variables. He also stated that he added 1 to satisfy the equation. This situation shows that the student's thinking in the question is ignored, and the problem itself is tried to be solved by focusing on the problem situation. Even though the written mathematical expression meets the form, it was evaluated as unsuitable because it needed to match the student explanations given in the problem situation.

Findings Obtained from Observation Data Regarding Specialized Content Knowledge (SCK) Dimensions

The observed SCK performance levels presented in Table 9 were derived through a structured analytical process. For each teacher, all 10 hours of observation transcripts were coded independently by two researchers using the subdimension-specific rubrics described in the Analysis of Data section (see Tables 3 and 4 for representative rubric criteria). Each observed instructional episode, defined as a discrete exchange or sequence of exchanges in which a teacher responded to a student question, evaluated a solution strategy, or employed a representation, was assigned one of the four performance levels: none (Level 0), inappropriate (Level 1), appropriate but incomplete (Level 2), or appropriate and complete (Level 3). The dominant level for each subdimension was determined by identifying the level that characterized the majority of coded episodes across the full 10-hour dataset for each teacher. Where a teacher's performance varied considerably across episodes within a subdimension, the overall classification reflected the most recurrent pattern rather than isolated instances of stronger or weaker performance. The summary labels in Table 9 therefore represent aggregate characterizations of each teacher's enacted SCK across the full observation period, not evaluations of individual lessons or moments. Teachers' 10-hour instructional processes were evaluated using analytical rubrics aligned with the three SCK subdimensions: justification, explanation, and representation.

Table 9. Summary of teachers observed SCK performance

SCK Component	T1	T2	T3	General Pattern
Justification	Appropriate, Incomplete	Mostly Appropriate, Complete	Appropriate, Incomplete	Conceptual gaps in evaluating student reasoning
Explanation	Appropriate, Complete	Appropriate, Incomplete	Appropriate, Incomplete	Preference for procedural shortcuts over conceptual depth
Representation	Strong (Numerical + Visual)	Strong (Numerical + Verbal)	Predominantly Numerical	Limited integration across representations

In the justification dimension, T2 demonstrated relatively stronger performance, whereas T1 and T3 frequently provided correct but incomplete evaluations of students' claims. In several cases, teachers recognized appropriate strategies but did not fully articulate the underlying mathematical structure, particularly when students questioned balance preservation or variable meaning. In the explanation dimension, T1's instructional explanations were consistently evaluated as appropriate and complete, showing effective connections between symbolic procedures and conceptual models (e.g., balance model). In contrast, T2 and T3 often preferred the inverse operation method as a practical shortcut and provided limited emphasis on the symmetric structure of equations. This procedural preference reduced the conceptual clarity of their explanations, despite their apparent subject knowledge. In the representation dimension, all teachers predominantly used numerical representations. T1 frequently integrated visual models (e.g., balance scales), while T2 relied more on verbal problem situations. T3 primarily focused on symbolic manipulation. Although representations were generally correct, cross-representation connections were not consistently emphasized across teachers. Overall, observation findings indicate that teachers' enacted SCK was strongest in procedural execution and weakest in conceptual elaboration and representation integration. These patterns largely align with the trends identified in the EKTT findings, suggesting consistency between teachers' test-based knowledge profiles and their classroom practices.

Subject-Specific SCK Duties and Responsibilities Derived from Observation Data

The duties and responsibilities associated with the justification, explanation, and representation dimensions of specialized content knowledge derived from the classroom observations are presented in Tables 10, 11, and 12, respectively. These duties and responsibilities were identified through an analysis of naturally occurring classroom interactions during equation instruction. Specifically, student questions, requests for clarification, misconceptions, and alternative solution attempts, together with teachers' responses to these situations, were examined. The resulting duties and responsibilities represent the types of specialized content knowledge that teachers were required to mobilize in order to respond effectively to students' mathematical thinking and instructional needs. Thus, the categories presented in the tables do not reflect predetermined competencies; rather, they emerged from the instructional demands observed during classroom practice. Collectively, these findings provide insight into the topic-specific forms of SCK that support effective teaching of first-order equations with one unknown.

Table 10. Duties and responsibilities of teachers regarding the justification dimension of specialized content knowledge in teaching first-order equations with one unknown

Ability to provide justifications regarding the meaning of the equality sign (interpreting it as an indicator of equivalence between expressions, not as a signal to produce a result)
Ability to provide justifications to prevent incorrect generalization of the balance model
Ability to provide justifications regarding the balancing both sides method
Ability to provide justifications that a given algebraic expression is not an operation that needs to be completed
Ability to provide justifications for alternative solution methods of equations
Ability to provide justifications for the application of the balance method
Ability to provide justifications for alternative solution methods to problems
Ability to provide justifications for why negative numbers cannot be used on the balance scale (because weights cannot be negative)
Ability to provide justifications for identifying unknowns and variables with the letter x
Ability to address the misconception of applying the inverse operation only to one side of the equation
Ability to provide justifications for why the same letters must take the same values in an equation
Ability to provide justifications that equations of the form ax+b=cx+d cannot be solved by reversing operations on coefficients
Ability to provide justifications for solving equations starting from the coefficient of the unknown
Ability to provide justifications for why the trial-and-error method will not always be effective

Ability to provide justifications regarding the meaning of the equality sign (interpreting it as an indicator of equivalence between expressions, not as a signal to produce a result)
Ability to address the misconception that different variables must take different values
Ability to provide justifications that the unknown is not necessarily located on only one side of the equation
Ability to provide justifications that unknowns are also variables

As shown in Table 10, the justification dimension required teachers to provide mathematical reasons for procedures, evaluate alternative strategies, and address common misconceptions related to equality, variables, and equation-solving methods. These responsibilities emerged when students questioned the validity of solution procedures or sought explanations for why particular mathematical actions were appropriate.

Table 11. Duties and responsibilities of teachers regarding the explanation dimension of specialized content knowledge in teaching first-order equations with one unknown

Being able to explain the definition of the concept of equality
Being able to explain the difference between the concepts of equality and equation
Being able to explain the use of the concept of equality as numerical sameness
Being able to explain the definition of the equation
Being able to explain why equation solving is learned
Being able to explain the application of the balance model
Being able to explain the balancing both sides method
Being able to explain that the inverse operation method is a shortcut to the balancing both sides method
Being able to explain how equation-solving steps connect to the balance method
Being able to explain how to find the unknown in an equation by modeling with a counting balance
Being able to explain the order of operations in solving equations
Being able to explain the relational meaning of the equality sign (equivalence between expressions, not an instruction to compute a result)
Being able to explain the definition of the concept of variable
Being able to explain the inverse operation method
Being able to explain the balancing both sides method in relation to the inverse operation method

Table 11 indicates that the explanation dimension primarily involved clarifying mathematical concepts, procedures, and relationships. The observed responsibilities highlight the need for teachers to make underlying mathematical ideas explicit and to connect procedural actions with conceptual understanding during equation instruction.

Table 12. Duties and responsibilities of teachers regarding the representation dimension of SCK in teaching first-order equations with one unknown

Being able to connect numerical representations with verbal representations (e.g., constructing real-life problems that represent equations, or establishing equations that represent real-life situations)
Being able to connect numerical representations with visual representations (e.g., showing equation-solving steps on a balance model or counting stamps)

Being able to connect numerical representations with verbal representations (e.g., constructing real-life problems that represent equations, or establishing equations that represent real-life situations)
Being able to connect verbal representations with visual representations (e.g., constructing a table or graph that represents a problem situation, or posing a problem based on a given table or graph)

As presented in Table 12, the representation dimension focused on establishing connections among verbal, numerical, and visual representations. These responsibilities emerged when teachers supported students in translating between different forms of representation and using them to reason about equations and problem situations.

Discussion and Conclusion

To examine the subdimensions of specialized content knowledge (SCK), participating teachers first completed the Equation Knowledge Test for Teaching (EKTT). Analysis of the justification dimension revealed that although teachers generally identified correct answers, their explanations lacked conceptual depth. Rather than analyzing why alternative strategies were mathematically valid or invalid, they tended to focus on procedural steps. This finding aligns with Li (2011), who reported that teachers' evaluations of equation solutions often remain operational rather than conceptual. Teachers frequently provided appropriate but incomplete justifications when evaluating students' alternative strategies and claims. While they could determine whether a method was correct, they struggled to articulate the underlying reasoning, particularly when addressing why a student's idea was incorrect. One contributing factor appears to be limited knowledge of common student misconceptions. Similar findings have been reported by Blömeke, Gustafsson, and Shavelson (2015) and other studies indicating that teachers' awareness of algebra-related misconceptions is often insufficient (Asquith et al., 2007; Baş, Erbaş, & Çetinkaya, 2011). These results support Ball et al.'s (2008) emphasis that SCK requires not only possessing conceptual knowledge but also enacting it in instructional practice, particularly in responding to student thinking.

Incomplete justifications were most evident in discussions related to the concepts of variable and unknown, the balancing both sides method, and the use of the balance model. Although teachers possessed procedural knowledge of these topics, they often used the concepts of variable and unknown interchangeably, leading to inconsistent explanations. As reported in prior research (Asquith et al., 2007; Huang & Kulm, 2012; İdil & Narlı, 2021), teachers may successfully perform algebraic procedures while lacking deep conceptual understanding of variable meaning. Such limitations influenced how they interpreted students' reasoning and, in some cases, contributed to persistent misconceptions in the classroom.

Another recurring pattern concerned teachers' preference for the inverse operation method. Although teachers were aware of the balancing both sides method and its structural logic, they emphasized procedural efficiency over conceptual transparency. Recognizing the equivalence between applying the same operation to both sides and using inverse operations is critical for structural understanding of equations (Qetrani, Ouailal, & Achtaich, 2021). The emphasis on rapid procedural solutions appeared to limit opportunities for students to develop a relational understanding of equality.

Findings from the explanation dimension further support this pattern. Although teachers knew the relevant rules and procedures, they often failed to provide explanations grounded in underlying mathematical meaning. For example, when asked whether transferring a term to the other side disturbed equality, two teachers provided rule-based responses without explicitly connecting the inverse method to the symmetric structure of equations. Richland, Stigler, and Holyoak (2012) similarly note that mathematics instruction frequently prioritizes procedural fluency over the conceptual structure that underlies it. Particular difficulty was observed in addressing the relational meaning of the equality symbol. Only one teacher consistently emphasized equality as equivalence between expressions rather than merely as a signal to perform operations. Prior research has shown that understanding equality relationally is foundational for algebraic reasoning (Knuth et al., 2008; Stephens et al., 2013; Van de Walle, Karp, & Bay-Williams, 2020). Limited attention to this relational meaning may contribute to students' misconceptions and procedural dependency.

In the representation dimension, teachers were generally able to express verbal situations symbolically. However, difficulties emerged when connecting numerical, visual, and verbal representations conceptually. Numerical representations predominated, while cross-representational transformations were limited. Similar findings have been reported by Asquith et al. (2007) and Mitchell, Charalambous, and Hill (2014), who note that insufficient coordination among representations constrains conceptual understanding.

Taken together, the findings indicate that teachers' SCK regarding first-order equations with one unknown is largely procedural in orientation. While teachers demonstrate operational fluency, the justification, relational, and representational dimensions of SCK remain insufficiently developed in classroom enactment. From an MKT perspective, this gap underscores the distinction between possessing mathematical knowledge and mobilizing that knowledge in ways that make structure visible to learners.

Suggestions

This study examined secondary school mathematics teachers' specialized content knowledge (SCK) regarding first-order equations with one unknown within the MKT framework and identified the SCK-related duties enacted in classroom practice. The findings indicate that teachers' limitations were concentrated in the justificatory, relational, and representational dimensions of SCK. Although procedural competence was generally evident, conceptual depth, particularly concerning the relational meaning of equality, the distinction between variable and unknown, and the structural equivalence of solution methods, remained underdeveloped. These results suggest that strengthening SCK requires more than reinforcing procedural fluency.

Teacher education programs should create structured opportunities for prospective teachers to interrogate the conceptual foundations of algebra, analyze student misconceptions, compare alternative solution strategies, and coordinate multiple representations. Practice-based coursework that situates algebraic structure within classroom interaction may provide a more robust pathway for developing enacted SCK. Similarly, in-service professional development should focus on transforming procedural knowledge into structurally grounded instructional explanations. Rather than prioritizing rapid solution techniques, professional learning initiatives should support teachers in unpacking the mathematical logic underlying equation-solving processes and in anticipating and responding to common student misconceptions. Engagement with current research and participation in professional learning communities may further sustain the development of SCK over time.

The findings underscore that the development of specialized content knowledge in algebra requires systematic attention to both mathematical structure and student reasoning. This study contributes to the MKT literature by demonstrating that the critical distinction in algebra teaching lies not merely in whether teachers possess mathematical knowledge, but in whether they are able to mobilize that knowledge in ways that render the structural foundations of algebraic reasoning transparent and accessible to students.

Future research should extend this type of integrated, topic-specific investigation to other algebraic concepts and grade levels and should explore the conditions under which in-service professional development most effectively supports the enactment of SCK in classroom practice. This study was produced from a doctoral dissertation completed at Dokuz Eylül University.

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