



Participatory Educational Research (PER)  
Vol.13(1), pp. 36-56, January 2026  
Available online at <http://www.perjournal.com>  
ISSN: 2148-6123  
<http://dx.doi.org/10.17275/per.26.03.13.1>

Id: 1749596

## An Examination of Middle School Mathematics Teachers' Pedagogical Content Knowledge Regarding Prime Numbers

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### Article history

#### Received:

23.06.2025

#### Received in revised form:

19.08.2025

#### Accepted:

20.09.2025

#### Key words:

middle school mathematics teachers, pedagogical content knowledge, prime numbers

This study examines middle school mathematics teachers' pedagogical content knowledge (PCK) regarding prime numbers. The study focuses on three subcomponents of PCK: Subject matter knowledge, knowledge of students' understanding, and knowledge of instructional strategies. A case study method, one of the qualitative research approaches, was employed. The participants consisted of 12 middle school mathematics teachers. Data were collected through the Prime Numbers Knowledge Test (PNKT) and semi-structured interviews developed by the researchers. The data were analyzed using content analysis. The findings revealed that mathematics teachers demonstrated limited PCK, particularly in the components of subject matter knowledge and knowledge of instructional strategies. Specifically, it was observed that teachers provided definitions and responded to questions related to prime numbers without including negative integers, indicating gaps in their subject matter knowledge. Regarding the knowledge of students' understanding component, participants showed satisfactory competence and performed better than in the other two components. Findings revealed that the teachers had a high level of awareness regarding students' learning difficulties, errors, and misconceptions, and they were able to correctly answer the questions posed by the researchers in this regard. Despite this relative strength, teachers' knowledge of instructional strategies was notably weak. Although teachers confused the concepts of strategy, method, and technique, they still applied these in their teaching practices. The presentation teaching strategy, the discovery-based teaching strategy, and the question-and-answer technique were identified as the most frequently implemented by the participants.

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## Introduction

Prime numbers are considered the building blocks of all number systems (Crilly, 2011). This statement can be considered a metaphorical interpretation of the fundamental theorem of arithmetic, which states that every positive integer can be decomposed into its prime factors and that this decomposition is unique (Zazkis & Liljedahl, 2004). It is seen that prime numbers are defined with different expressions. For example, there are definitions such as “integers greater than 1 with no positive divisor other than 1 and itself”, “natural numbers greater than 1 that cannot be written as the product of two smaller natural numbers”, and “positive integers with only four integer divisors” (Gardner, 1984). At this point, it is crucial to correctly state the essential properties that determine prime numbers: The number is greater than 1 and has two positive divisors.

Students' knowledge and readiness levels should be considered when using these definitions. For example, 6th-grade students do not know the concepts of positive and negative since they have not yet learned the subject of integers. For this reason, a definition can be made using the concept of natural numbers. On the other hand, the fundamental theorem of arithmetic should be examined to understand why 0, 1, and negative numbers are not prime. The fundamental theorem of arithmetic: Let  $n > 1$  be an integer. Then there is such an integer  $r > 1$  and such prime numbers  $p_1, p_2, p_3, \dots, p_r$  that  $n = p_1 \cdot p_2 \cdot p_3 \cdot \dots \cdot p_r$ , and this representation is unique with the difference of changing the places of the factors (Baştan, 2019). When examined with an example of why 1 is not included in the definition of prime number, it can be written as the multiplication of prime numbers in infinite ways. Therefore, it creates a situation contrary to the fundamental theorem of arithmetic. A similar situation applies to discussing the primality of 0 and negative integers.

Research on prime numbers shows that students have various misconceptions and learning difficulties. For example, Özdeş (2013) stated that it is common to accept 0 and 1 as prime numbers and to think that negative numbers can be prime. On the other hand, Zazkis and Campbell (1996) found that students think that big numbers should have multipliers and frequently make conceptual errors, such as that prime numbers are only small numbers. Akkan and Öztürk (2019) drew attention to the misconception that 2 is not accepted as prime and all odd numbers are perceived as prime. In addition, Zazkis and Liljedahl (2004) emphasized common errors such as the belief that the product of two prime numbers is also prime and checking only divisibility by 2, 3, and 5 when evaluating whether a number is prime.

Various misconceptions and learning difficulties related to the subject of prime numbers necessitate the creation of an effective learning environment. In this regard, Baştürk Şahin et al. (2017) implemented an instructional intervention based on the theory of didactic situations to facilitate the acquisition of the concept of prime numbers and examined the outcomes in terms of improving instruction. The findings indicated that instructional practices aligned with the phases of the theory, in accordance with the principles of constructivism, were suitable for teaching the concept of prime numbers, and that lessons had positive effects on students. Similarly, Yağmur (2020) designed and examined a game for sixth-grade students on the topic of factors and multiples, aiming to create a differentiated and enriched learning environment for the curriculum objective of identifying prime numbers and using the Sieve of Eratosthenes to determine prime numbers up to 100. Observations and student feedback revealed that the game was both enjoyable and educational, increasing students' motivation and willingness to participate. In another study, Ceylan Oral (2021) evaluated the classroom applicability of the Prime Factor Board (Asal Çarpan Kartelası) developed as an alternative teaching material for

the “factors and multiples” unit in middle school mathematics. Both teacher and student responses suggested that the material was simple, clear, aligned with learning objectives, and accessible for use. Students described lessons using the material as exciting, engaging, instructive, useful, and enjoyable.

An effective learning environment can be created by teachers who are competent in their field and have pedagogical knowledge (Putnam et al., 1992). This situation reveals the importance of the Pedagogical Content Knowledge (PCK) concept, which includes both subject matter and pedagogical knowledge (Yurtyapan, 2018). Shulman (1986) defined PCK as the ability to plan appropriately, design instructional strategies, and implement them to ensure effective teaching of the subject matter. He also emphasized that a single method is not always the most effective in teaching a subject and that teachers should know various teaching methods. In addition, he also pointed out the importance of teachers shaping their teaching by taking students' prior knowledge into consideration.

This study examined mathematics teachers' PCK regarding prime numbers. The theoretical framework proposed by Gökbulut (2010) was utilized in this process. Gökbulut (2010) based on the models developed by different researchers (Cochran et al., 1993; Grossman, 1990; Marks, 1990; Shulman, 1986) while examining preservice teachers' PCK. In his study, the components of PCK were subject matter knowledge, knowledge of students' understanding, knowledge of instructional strategies, and curriculum knowledge. Curriculum knowledge was not included in the components addressed in this study. The reason is that prime numbers are covered only at the 6th-grade level, with two related objectives (MoNE, 2018).

In the literature, various studies have been conducted to examine teachers' pedagogical content knowledge (PCK) in many different subjects, such as the concept of numbers, average, ratio-proportion, fractions, triangles, and quadrilaterals (Doğruel, 2019; Huang et al., 2025; Li et al., 2025; Satan et al., 2024; Tossavainen, 2025; Yurtyapan, 2018; Watson & Callinham, 2013). However, no study has been found that examines teachers' pedagogical content knowledge (PCK) regarding prime numbers and topics related to prime numbers (such as prime factors, divisibility, etc.). Therefore, there is a lack of information about teachers' conceptual and instructional knowledge as well as their classroom practices related to this topic. Accordingly, this study not only fills a significant gap in the literature on prime numbers but also reveals the knowledge and skills that middle school mathematics teachers and prospective teachers need in teaching prime numbers. It will directly contribute to the content and approach of teacher education programs. The findings will provide concrete guidance for teacher educators in lesson design, instructional strategies, and planning professional development activities.

This study examines middle school mathematics teachers' PCK regarding prime numbers in the subcomponents of subject matter knowledge, knowledge of students' understanding, and knowledge of instructional strategies. In line with this purpose, the sub-questions of the study were determined as follows:

- (1) How is middle school mathematics teachers' subject matter knowledge related to prime numbers?
- (2) How is middle school mathematics teachers' knowledge of students' understanding related to prime numbers?
- (3) How is middle school mathematics teachers' knowledge of instructional strategies related to prime numbers?

## Method

### *Research design*

In this study, which aims to examine middle school mathematics teachers' PCK regarding prime numbers in detail, the case study design, one of the qualitative research methods, was used. The case study is a research method that aims to explore a situation in depth with a limited number of participants and utilizes different data collection tools (McMillan & Schumacher, 2010).

### *Working group*

This study was carried out during the 2023–2024 academic year with a sample of 12 middle school mathematics teachers employed at public schools located in a province within the Southeastern Anatolia Region. Teachers participated in the study voluntarily. Their actual names were not used, and they were assigned codes such as T1, T2, T3, ..., and T12. Convenience sampling was employed due to the teachers' demanding workload and limited time availability. Information about the teachers is presented in Table 1.

Table 1. Demographic information of teachers

Teacher	Gender	Professional Experience	Educational Level
T1	Female	2 Years	Undergraduate degree
T2	Female	3 Years	Undergraduate degree
T3	Female	4 Years	Undergraduate degree
T4	Female	5 Years	Undergraduate degree
T5	Female	5 Years	Undergraduate degree
T6	Female	7 Years	Graduate degree
T7	Male	8 Years	Undergraduate degree
T8	Female	9 Years	Undergraduate degree
T9	Female	10 Years	Undergraduate degree
T10	Female	11 Years	Graduate degree
T11	Male	11 Years	Graduate degree
T12	Male	13 Years	Undergraduate degree

### *Data collection tools*

Data for this study were gathered using the Prime Number Knowledge Test (PNKT) alongside semi-structured interviews. The researchers first examined the goals and objectives in the curriculum, middle school mathematics textbooks, and the related literature to determine students' common errors and misconceptions, and the knowledge levels that teachers should have. In this direction, the knowledge levels that should be measured in terms of subject matter knowledge, knowledge of students' understanding, and knowledge of instructional strategies, which are the subcomponents of PCK, were determined (Akkan & Öztürk, 2019; Ercire, 2022; MoNE, 2018; Özçelik, 2023; Özdeş, 2013). Table 2 shows the knowledge levels to be examined in the subcomponents of PCK.

Table 2. Knowledge levels of PCK components

PCK Component	Knowledge Level
Subject Matter Knowledge	<ol style="list-style-type: none"> <li>1. Being able to use the definitions and concepts related to prime numbers correctly</li> <li>2. Being able to give correct answers to students' questions and explain the reasons</li> <li>3. Being able to explain the importance, goals, and objectives of prime numbers</li> </ol>
Knowledge of Students' Understanding	<ol style="list-style-type: none"> <li>1. Being able to identify students' learning difficulties, errors, and misconceptions related to prime numbers</li> <li>2. Being able to explain the reasons for students' learning difficulties, errors, and misconceptions related to prime numbers</li> </ol>
Knowledge of Instructional Strategies	<ol style="list-style-type: none"> <li>1. Eliminating students' learning difficulties, errors, and misconceptions related to prime numbers</li> <li>2. Knowing appropriate strategies, methods, and techniques related to prime numbers and being able to apply this knowledge of strategies, methods, and techniques in teaching</li> </ol>

PNKT, consisting of 10 questions, was prepared according to the knowledge levels determined in Table 2. The test was reviewed by three experts in mathematics education. Experts suggested that the current order of the questions is complicated, so the questions should be organized according to knowledge levels. Also, it would be appropriate to add knowledge-based questions first and then scenario-based questions. On the other hand, two instructors suggested that one question should be removed from the test, stating that it did not measure subject matter knowledge and was related to curriculum knowledge. In addition, the phrase “Which methods/techniques do you use?” was found to be ambiguous, and it was suggested to clarify it with examples like “direct instruction” or “discovery-based teaching”. Based on the suggestions, the test was revised and resent to the experts for final review.

The first question in the PNKT addresses the importance of prime numbers, while the second question examines how prime numbers are defined and the changes in the definitions used according to grade levels. Regarding the third question, teachers were asked to explain the primality of 0, 1, and negative numbers, which students made the most mistakes on and asked about frequently. The fourth question measured teachers' teaching skills by determining students' readiness levels. The fifth question aimed to identify the methods and techniques teachers use when teaching prime numbers. The sixth question, which was prepared as a scenario, explored how teachers explained that only 2 is a prime even number and how they presented this to students. The seventh question, which was also prepared as a scenario, explored teachers' views on why students misidentified the smallest prime number and how they would address the mistake. In the eighth question, similar to the questions in the textbooks, how the teachers explained the question to the students, how they identified the mistakes that the students might make, and how they would correct these mistakes were revealed. The ninth question examined how teachers used the Sieve of Eratosthenes and presented it to students, and how students answered the questions. Table 3 illustrates the knowledge levels that the questions composing the PNKT are designed to assess. The questions in the PNKT are coded as “Q1, Q2, Q3, ..., Q9”, and the sub-questions are coded as “A, B, and C.” On the other hand, the knowledge levels that the questions and sub-questions are intended to assess are presented in three levels: “1, 2, and 3”.

Table 3. Knowledge levels aimed to be measured in the questions composing the PNKT

PCK Component	Q1	Q2	Q3	Q4	Q5	Q6	Q7			Q8			Q9	
							A	B	C	A	B	C	A	B
Subject Matter Knowledge	3	1	1			2				1			2	1
Knowledge of Students' Understanding				1			2				1, 2			
Knowledge of Instructional Strategies			2	1	2			1	2				1	

Semi-structured interviews were held to complement PNKT data and explore teachers' deeper knowledge of prime numbers. The interview questions were developed using Akkaş's (2014) study. In the interview consisting of eight questions, teachers were asked questions about the methods and strategies they used while teaching the subject of prime numbers, students' participation in the teaching process, students' learning difficulties, errors and misconceptions, and their reasons, feedback given to students, and activities carried out during the teaching process. Teachers' approaches to prime numbers and solutions to students' difficulties, errors, and misconceptions were also discussed.

### *Data collection process*

The study data were collected over three weeks. Middle school mathematics teachers were given information about the purpose, method, and tools of the study, and were assured that the data obtained would remain confidential. Semi-structured interviews were first conducted at the participants' schools. Interviews with each participant lasted an average of 30 minutes. Participants were encouraged to feel comfortable and respond thoughtfully during the study. All interviews were conducted one-on-one and audio recorded. Following the interviews, the teachers were given the PNKT and asked to answer the questions. Teachers completed the PNKT in an average of 90 minutes.

### *Data analysis*

Content analysis was used to examine data from the interviews and PNKT to reveal middle school mathematics teachers' PCK regarding prime numbers. Content analysis is a systematic technique for summarizing similar data into smaller categories (Büyüköztürk et al., 2014). It involves detailed data review, creation of categories, identification of emerging codes, and interpretation of their relationships (Ercire, 2022). The data obtained from the PNKT and semi-structured interviews were analyzed in three sub-dimensions: Subject matter knowledge, knowledge of students' understanding, and knowledge of instructional strategies. Before analyzing the data, word-for-word transcripts of the interview records were created.

Teachers' responses to the questions about subject matter knowledge were coded at three levels: "improved", "open to improvement", and "should be improved". The codes and explanations related to the codes are presented in Table 4.

Table 4. Codes and explanations of codes related to subject matter knowledge

Codes	Explanations of Codes
Improved	Has in-depth knowledge of the topic and presents original and effective ideas.
Open to Improvement	Knows the topic's essential dynamics but cannot analyze it in detail.
Should Be Improved	Demonstrates a superficial approach to the topic and does not have sufficient knowledge.

Teachers' knowledge of students' understanding was addressed in two categories: “being able to identify students' learning difficulties, errors, and misconceptions” and “being able to identify the reasons for students' learning difficulties, errors, and misconceptions”. The codes related to these categories are given in Table 5.

Table 5. Categories and codes related to the knowledge of students' understanding

Categories	Codes
Being able to identify students' learning difficulties, errors, and misconceptions	Accepting the number 1 as prime
	Accepting all odd numbers as prime numbers
	Thinking that the number 2 is not prime
	Not being able to establish the relationship between even numbers and primality
	Memorization
	Not being able to find the divisors of numbers
Being able to explain the reasons for students' learning difficulties, errors, and misconceptions	Over-generalization
	Lack of knowledge
	Over-rule
	Carelessness

The third component, teachers' knowledge of instructional strategies, was categorized into two categories: “eliminating students' learning difficulties, errors, and misconceptions” and “knowing appropriate strategies, methods, and techniques related to the subject and being able to apply this knowledge in the teaching”. The codes related to these categories are given in Table 6.

Table 6. Categories and codes related to the knowledge of instructional strategies

Categories	Codes
Eliminating students' learning difficulties, errors, and misconceptions	Giving information and saying the correct answer
	Solving the problem together
	Explaining by giving examples
	Helping him/her find his/her own mistake
	Presentation teaching strategy
	Direct instruction method
Knowing appropriate strategies, methods, and techniques related to the subject and being able to apply this knowledge in teaching	Question-and-answer technique
	Sample question solution
	Discovery-based teaching strategy
	Drama technique
	Activity-based approach

## Validity and reliability

To ensure the validity and reliability of the study, several measures were taken throughout the research process. PNKT and the semi-structured interview form were developed and revised based on expert feedback. To enhance reliability, three researchers independently analyzed the data, and consensus was reached through comparison to establish common codes and categories. Direct quotations were included to support the credibility of the findings and reflect participants' views accurately.

To strengthen validity, the research design, participant characteristics, and data collection and analysis procedures were described in detail. The findings were presented systematically to allow comparisons with similar contexts, and the overall process was reported transparently to support the study's transferability.

## Findings

Middle school mathematics teachers' PCK regarding prime numbers was examined in terms of subject matter knowledge, knowledge of students' understanding, and knowledge of instructional strategies. The findings are presented under these three categories.

### *Findings related to mathematics teachers' subject matter knowledge*

This section addresses mathematics teachers' subject matter knowledge related to prime numbers. In this context, teachers' abilities to use definitions and concepts related to prime numbers correctly, provide accurate and reasoned answers to students' questions, and explain the importance, objectives, and goals of the subject were examined.

The findings obtained from teachers' responses to questions about how they define prime numbers are summarized in Table 7.

Table 7. Teachers' definitions of prime numbers

Codes	Teacher(s)	Examples
Improved	T6, T9, T12	<i>T12: Natural numbers greater than 1 that have no positive divisors other than 1 and themselves.</i>
Open to Improvement	T3, T4, T5, T8, T11	<i>T11: Numbers greater than 1 that have no divisors other than 1 and themselves are called prime numbers.</i>
Should Be Improved	T1, T2, T7, T10	<i>T7: Numbers that have no divisors other than 1 and themselves are called prime numbers.</i>

Critical points considered in defining prime numbers were having two positive divisors and being greater than 1. As shown in Table 7, since the definitions provided by most teachers include only one of the critical components, they were categorized as open to improvement, while the definitions given by four teachers were classified as should be improved. Three teachers took the critical components into account and gave improved definitions of prime numbers. For instance, T12 provided an improved response by stating that prime numbers have exactly two positive divisors and that they are greater than 1. T11, who provided a definition categorized as open to improvement, stated that prime numbers are greater than 1 but did not mention that they have exactly two positive divisors. On the other hand, T7 provided a response categorized as should be improved, as their definition did not include that prime numbers have exactly two positive divisors, namely 1 and the number itself, and that they are greater than 1.

The findings obtained from the teachers' responses to the question of whether 0 is a prime number are presented in Table 8.

Table 8. Teachers' explanations regarding why 0 is not a prime number

Codes	Teacher(s)	Examples
Improved	T6, T7, T10, T11, T12	T10: By definition, prime numbers are positive and have only two divisors, so I would say that 0 (excluding 0) is divisible by every number.
Open to Improvement	T2, T9	T9: Since 0 is divisible by every number, it cannot be a prime number.
Should Be Improved	T1, T3, T4, T5, T8	T5: When we examine the divisors of 0, we see that it has only one divisor, 0. For a number to be prime, it must have at least two divisors, one of which must be 1.

Table 8 shows that most of the teachers' explanations for why zero is not considered a prime number were evaluated as either improved or should be improved. T10 provided an improved answer by stating that zero does not meet the definition of a prime number because it has more than two divisors. T9, who provided a response categorized as open to improvement, stated that zero is divisible by every number. However, T9 did not mention that division by 0 itself is undefined. On the other hand, T5 gave an incorrect response by stating that zero has only one divisor, which is zero itself, and this response was categorized as should be improved.

The findings obtained from teachers' responses to the question of whether 1 is a prime number are presented in Table 9.

Table 9. Teachers' explanations regarding why 1 is not a prime number

Codes	Teacher(s)	Examples
Improved	T4, T6, T7, T8, T11, T12	T7: 1 is not prime by definition. Natural numbers greater than 1 are prime numbers. Furthermore, we can write all natural numbers as the product of their prime factors. If 1 were prime, we could derive it in many ways. $15=1.3.5=1^2.3.5$ .
Open to Improvement	T3, T9	T3: The number 1 is not considered a prime number because it does not lead us to a result when dividing other numbers into their prime factors.
Should Be Improved	T1, T2, T5, T10	T10: By definition, prime numbers are positive and have only two divisors, while 1 has only one divisor.

As shown in Table 9, most teachers' explanations regarding why 1 is not a prime number were categorized as improved, while four teachers' responses were classified as should be improved. For example, T7 correctly explained that 1 is not a prime number by utilizing both the fundamental theorem of arithmetic and the definition of prime numbers, providing examples to support the explanation. The response provided by T3 regarding the fundamental theorem of arithmetic is unclear. On the other hand, the number 1 has two divisors: 1 and -1. Therefore, T10's definition was classified as should be improved, as it did not specify that only positive divisors should be considered in the definition of prime numbers.

The findings obtained from teachers' responses to the question of whether negative numbers are prime numbers are presented in Table 10.

Table 10. Teachers' explanations regarding why negative numbers are not prime

Codes	Teacher(s)	Examples
Improved	T3, T5, T6, T11, T12	<i>T3: In negative numbers, for example, <math>10=2.5=-2.-5</math> is expressed in two different ways.</i>
Open to Improvement	T2, T10	<i>T2: We do not include negative numbers in prime numbers. I am stating the rule as it is.</i>
Should Be Improved	T1, T4, T7, T8, T9	<i>T9: Negative numbers cannot be prime because they have a divisor other than 1 and themselves, namely -1.</i>

According to Table 10, five teachers provided improved explanations regarding why negative numbers are not prime, while the explanations of five teachers were categorized as should be improved. Additionally, the responses of two teachers were found to be lacking in information and were classified as open to improvement. T3, who provided an improved response, stated based on the fundamental theorem of arithmetic that every positive integer can be uniquely expressed as a product of prime numbers, and accordingly explained that negative numbers cannot be prime. T2 stated that negative numbers are excluded from prime numbers by rule. Since the teacher did not explain what the rule is, the explanation was categorized as open to improvement. T9 stated that negative numbers are not prime because they have divisors other than 1 and themselves. However, since the explanation lacked the information that prime numbers must have exactly two positive divisors and be greater than 1, the response was categorized as should be improved.

In order to analyze the teachers' subject matter knowledge, they were presented with a problem type frequently used during instruction: "What values can A take for the number  $7A$  to be a two-digit prime number?" The teachers' solution strategies for this problem were examined, and the findings from their responses are presented in Table 11.

Table 11. Teachers' responses regarding the primality of the two-digit number  $7a$ 

Codes	Teacher(s)	Examples
Improved	T3, T4, T5, T6, T8, T12	<i>T8: Instead of the number A, the even numbers 0, 2, 4, 6, and 8 cannot be used, 0 and 5 cannot be used because they are divisible by 5, and 7 cannot be used because it is divisible by 7.</i>
Open to Improvement	T2, T7, T10	<i>T7: There are 10 numbers that A can take, and I try them all one by one.</i>
Should Be Improved	T1, T9, T11	<i>T9: <math>7A</math>; <math>A=1, 3, 9</math>, so <math>1+3+9=13</math>.</i>

Table 11 reveals that six teachers provided improved responses, while the number of teachers who gave responses categorized as open to improvement and should be improved was equal. For example, T8, who gave an improved response, eliminated the digits that A could not take based on divisibility rules. T7, who gave an open to improvement response, stated that they would try each possible digit for A one by one. However, it was observed that T7 did not apply any elimination based on divisibility rules. T9, on the other hand, directly found the correct answer through calculation. However, since T9 did not explain why 1, 3, and 9 are valid digits for A or why the other digits are not acceptable, his/her response was coded as should be improved.

Another aspect of subject matter knowledge examined was teachers' ability to answer students' questions and explain their reasoning. Teachers were asked to respond to likely student questions.

Most teachers stated in interviews that they use the Sieve of Eratosthenes, as presented in the 6th-grade textbook, as a classroom activity. They were asked student-like questions such as whether it can be used for numbers beyond 100 or if checking the primality of 283 requires testing divisibility by all numbers up to 283. Teachers' responses on using the griddle and identifying large prime numbers are shown in Table 12.

Table 12. Teachers' knowledge regarding the use of the Sieve of Eratosthenes for large numbers

Codes	Teacher(s)	Examples
Improved	T4, T9, T11, T12	<i>T4: It is sufficient to check whether the number 283 is divisible by prime numbers up to 13. <math>16 &lt; \sqrt{283} &lt; 17</math>, primes smaller than 17 are checked. The numbers 2, 3, 5, 7, 11, and 13 are checked.</i>
Open to Improvement	T1, T2, T3, T6, T7, T8, T10	<i>T1: I would try the numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. If none of these divide the number, I would conclude that it is prime.</i>
Should Be Improved	T5	<i>T5: I would say that looking at the first 10 numbers is sufficient. I would emphasize that if they want to continue, they can use the divisibility rules.</i>

The findings show that seven teachers gave responses coded as open to improvement, and one teacher gave a response coded as should be improved. Four teachers gave improved responses by stating that a number's divisibility should be checked using prime numbers smaller than its square root. For example, T4, who gave an improved response, demonstrated knowledge that 283 should be tested using prime numbers smaller than its square root and correctly identified which primes to use. T1, with an open to improvement response, understood the need to check prime factors but was unsure how far to go. T5, whose response was coded as should be improved, incorrectly stated that checking the first ten numbers is sufficient, despite 221 being a non-prime whose divisors are not among the first ten primes.

One of the most common mistakes students make is the misconception regarding whether the number 2 is a prime number. When teachers state that all even numbers are not prime because they are divisible by 2, students tend to assume that 2 is also not a prime number, since it is even. In this context, the responses given by teachers to correct this misconception among students are presented in Table 13.

Table 13. Teachers' responses regarding the primality of the number 2

Codes	Teacher(s)	Examples
Improved	T3, T8, T12	<i>T8: Prime numbers have no positive divisors other than 1 and themselves. The positive divisors of the number 2 are 1 and 2. However, other even numbers are not prime numbers because they are divisible by 2 in addition to themselves and 1. For example, the divisors of 10 are 1, 2, 5, and 10. They are not prime numbers since they have divisors other than 1 and themselves.</i>
Open to Improvement	T1, T2, T4, T5, T6, T7, T9, T10, T11	<i>T4: Prime numbers are positive numbers that can only be divided by 1 and themselves. The number 2 is prime because it can be divided by 1 and 2. For example, the number 16 is not prime because it can be divided by 1, 2, 4, 8, and 16.</i>
Should Be Improved	-	-

As seen in Table 13, nine teachers provided open to improvement responses, while three gave improved responses. No teacher provided a response coded as should be improved. For instance, T8, who gave an improved response, correctly explained that 2 is prime because it has

exactly two positive divisors, 1 and itself, and noted that other even numbers do not meet this definition. T4, with an open to improvement response, stated that 2 is prime but explained why other even numbers are not using only the example of 16. While this helps students understand that 16 is not a prime number, it does not enable them to generalize this understanding to other even numbers.

Another subtopic assessing subject matter knowledge was teachers' perceptions regarding the importance, objectives, and goals of teaching prime numbers. Their views are summarized in Table 14.

Table 14. Teachers' explanations regarding the importance, objectives, and goals of prime numbers

Codes	Teacher(s)	Examples
Improved	T9, T12	<i>T9: Yes, every whole number greater than 1 is either a prime number or a product of prime numbers. Prime numbers form the basis of encryption. If we want to encrypt huge numbers, we can generate the numbers using prime numbers.</i>
Open to Improvement	T3, T4, T5, T6, T7, T8, T10, T11	<i>T5: Yes, it is essential. Without the concept of prime numbers, it would be difficult to understand or reach other natural numbers. Every number is formed by multiplying prime numbers with each other and themselves.</i>
Should Be Improved	T1, T2	<i>T1: Yes, it is essential. Division is one of the most frequently used operations in daily life. Knowing that prime numbers cannot be divided by any number other than 1 and themselves not only provides convenience but also makes these special numbers intriguing to people.</i>

According to Table 14, eight teachers provided open to improvement responses, while two teachers gave improved responses, and another two gave responses entailing improvement. T9, who gave an improved response, accurately and fully stated the fundamental theorem of arithmetic. T9 also mentioned the relationship between prime numbers and encryption. T5 provided an open to improvement response by giving incomplete information regarding the fundamental theorem of arithmetic. T1, who gave a should be improved response, focused less on the role and importance of prime numbers in mathematics and instead related them to the previous topic, providing information about how to find prime numbers.

### ***Findings related to mathematics teachers' knowledge of students' understanding***

In examining mathematics teachers' knowledge of students' understanding of prime numbers, their ability to identify students' learning difficulties, errors, and misconceptions, as well as to explain the underlying reasons, was considered.

According to the teachers, students' learning difficulties, errors, and misconceptions related to prime numbers are presented in Table 15.

Table 15. Learning difficulties, errors, and misconceptions are identified by teachers in students

Codes	Teacher(s)	Examples
Accepting the number 1 as prime	T1, T3, T5, T6, T7, T9, T12	<i>T12: We encounter questions such as “Why is 1 not a prime number? It is divisible only by 1 and itself, which fits the definition.”</i>
Accepting all odd numbers as prime numbers	T3, T4, T5, T8, T10	<i>T4: One of the most common mistakes is that after learning that there are no prime numbers other than two in even numbers, students misunderstand that all prime numbers are odd numbers.</i>
Thinking that the number 2 is not prime	T9, T10, T12	<i>T9: They forget about 2. They think that even numbers are not prime numbers and say that 2 is not a prime number.</i>
Not being able to establish the relationship between even numbers and primality	T2, T6, T9	<i>T2: There are no even prime numbers other than two; it is unique and special. Despite my saying this, they are trying to examine even numbers. They are not using that short method.</i>
Memorization	T1, T11	<i>T11: When listing prime numbers, we say 2, 3, 5, 7, and then 11 and 13. While counting this way, students tend to think of it like rhythmic counting and may mistakenly assume that 9 is also a prime number.</i>
Not being able to find the divisors of numbers	T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12	<i>T7: Children have deficiencies; they cannot find the factors of numbers. Therefore, they cannot decide whether a number is prime or not.</i>

As seen in Table 15, teachers identified multiple learning difficulties, errors, and misconceptions among students. A common issue raised by seven teachers was that students often consider the number 1 as a prime number. For instance, T12 stated that students asked, “Why is 1 not a prime number? It is divisible only by 1 and itself, which fits the definition”. Additionally, five teachers pointed out that students tend to accept all odd numbers as prime numbers. According to T4, this misconception stems from instruction emphasizing that no even number other than 2 is prime, and students overgeneralize and assume that all odd numbers must be prime. Another reported issue was the belief that 2 is not a prime number, mentioned by three teachers. T9 stated that students forget about 2 and assume that even numbers are not prime numbers, which leads them to incorrectly conclude that 2 is not a prime number. In relation to this, three teachers noted that students often fail to connect even numbers and the concept of primality. T2 shared that even though it is emphasized that 2 is unique and the only even prime number, students still continue to examine other even numbers, indicating that they are not applying this short method. Two teachers linked students’ errors to memorization. T11 noted that students sometimes treat prime numbers as following a pattern, which leads them to mistakenly consider 9 as prime. Lastly, all of the teachers emphasized that students struggle to find the divisors of numbers, which directly affects their ability to determine whether a number is prime. T7 pointed out that “children have deficiencies; they cannot find the factors of numbers. Therefore, they cannot decide whether a number is prime or not.”

Students may have multiple learning difficulties, errors, and misconceptions. According to their teachers, the reasons behind these issues and the corresponding explanations are presented in Table 16.

Table 16. Causes of students' learning difficulties, errors, and misconceptions according to teachers

Codes	Teacher(s)	Examples
Over-Generalization	T1, T3, T4, T5, T7, T8, T10, T11, T12	<i>T7: The reason students do not accept 2 as a prime number is that when the statement "all even numbers are not prime" is used, they conclude that since 2 is even, it cannot be prime either.</i>
Lack of Knowledge	T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12	<i>T2: Students have difficulties finding the divisors of numbers and determining whether a number is prime. They lack understanding of divisibility rules and cannot identify which numbers divide the given number and therefore cannot determine its primality.</i>
Over-Rule	T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12	<i>T11: The reason students consider all odd numbers as prime is that when the statement "all even numbers except 2 are not prime" is used, they think, "If even numbers are not prime, then all odd numbers must be prime."</i>
Carelessness	T2, T6, T8, T9, T10	<i>T6: The reason students start listing prime numbers as 2, 3, 5, 7 and then also consider 9 as prime. 3, 5, and 7, followed by 11 and 13, so they mistakenly assume that 9 fits as a prime number in this pattern.</i>

An examination of Table 16 reveals that all teachers agreed on lack of knowledge and over-rule as the primary reasons behind students' learning difficulties, errors, and misconceptions. Additionally, a considerable number of teachers also identified over-generalization as a significant factor. For instance, T7 emphasized over-generalization, noting that when students are taught "even numbers are not prime," they tend to conclude that "2 is even and therefore cannot be prime." Addressing lack of knowledge, T2 pointed out that students struggle to identify the divisors of numbers and, as a result, are unable to determine whether a number is prime. In relation to over-rule, T11 explained that when students are told "all even numbers except 2 are not prime," they tend to generalize this rule and assume that all odd numbers must be prime. One of the five teachers who highlighted carelessness, T6, stated that students often list prime numbers as "2, 3, 5, 7, 9, 11, 13," carelessly including 9 in the sequence.

### ***Findings related to mathematics teachers' knowledge of instructional strategies***

In examining teachers' knowledge of instructional strategies, the focus was on both eliminating students' learning difficulties, errors, and misconceptions about prime numbers, and understanding as well as applying appropriate strategies, methods, and techniques for teaching the topic effectively.

The findings regarding how teachers addressed the learning difficulties, errors, and misconceptions they encountered in students are presented in Table 17.

Table 17. Teachers' addressing of students' learning difficulties, errors, and misconceptions

Codes	Teacher(s)	Examples
Giving information and saying the correct answer	T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12	T11: To prevent misconceptions, I focus on the definition. I give the answer by providing both the definition and the necessary information.  T8: I provide the information needed based on the question. For example, if a student cannot determine whether 17 is a prime number, I give the definition of a prime number and then ask guiding questions such as 'Does 17 have any divisors other than 1 and itself?' so that the student can figure it out on their own.
Solving the problem together	T2, T5, T7, T8, T12	T6: I try to encourage them to think by providing counterexamples. For instance, if a student says that even numbers other than 2 are prime, I respond by saying, 'Let's find their divisors,' and guide them to reconsider the definition through examples.
Explaining by giving examples	T1, T3, T4, T6, T8, T10, T11, T12	T2: I ask them to explain why they gave that answer. By giving some hints, I help them realize their mistake and guide them toward the correct answer.
Helping him/her find his/her own mistake	T2, T5	

As seen in Table 17, all teachers reported supporting students by providing information and giving the correct answer when addressing their learning difficulties, errors, and misconceptions. Additionally, five teachers mentioned solving problems together with students, eight used examples in their explanations, and two helped students identify their own mistakes. For example, T11 corrected students' incorrect responses by reminding them of the definition of a prime number. T8 solved the question with students, provided necessary information, and asked guiding questions. T6 used counterexamples to help students recognize their mistakes and reconsider the definition through examples. T2 encouraged students to explain their reasoning and provided hints to help them identify and correct their errors.

The findings on the strategies, methods, and techniques that teachers employ when teaching prime numbers are summarized in Table 18.

Table 18. Strategies, methods, and techniques used by teachers in their lessons

Codes	Teacher(s)	Examples
Presentation teaching strategy	T1, T3, T4, T7, T9, T12	T3: I use both the presentation and the discovery-based teaching. Due to students' readiness levels and the large class size, I use presentation.
Direct instruction method	T1, T7, T8, T11	T1: I use the direct instruction method.
Question-and-answer technique	T2, T3, T4, T7, T8, T9, T12	T4: I generally use the discovery-based teaching method and the question-and-answer method.
Sample question solution	T1, T3, T7, T10	T7: I have students examine the factors of natural numbers one by one. I explain that numbers that have only 1 and themselves as factors are called prime numbers.
Discovery-based teaching strategy	T3, T4, T5, T6, T7, T8, T9, T12	T5: I use the discovery-based teaching method. I have students examine the factors of natural numbers one by one. I explain that numbers with only 1 and themselves as factors are called prime numbers.

Drama technique	T9	<i>T9: I ask students to put themselves in the place of prime numbers and to imagine placing their factors in their pockets. Each student notices that a prime number has only two factors. They also observe that non-prime numbers have more than two divisors.</i>
Activity-based approach	T6, T9	<i>T6: We find prime numbers through activities using the Sieve of Eratosthenes.</i>

According to Table 18, teachers employ multiple strategies, methods, and techniques in their lessons, with most preferring the discovery-based teaching strategy, the question-and-answer technique, and the presentation strategy. For example, T5 stated that he/she tries to help students understand the concept of prime numbers through their own discovery process by using the discovery-based teaching strategy. T4 mentioned that he/she generally uses the discovery-based teaching strategy and the question-and-answer technique to explain prime numbers. T3 indicated that, due to students' readiness levels and the large class size, he/she uses the presentation teaching strategy. It was also observed that teachers used the direct instruction method and the sample question solution approach. For example, while T1 stated that he/she uses the direct instruction method, T7 mentioned that he/she has students examine the factors of natural numbers one by one and explained that those meeting the conditions are prime numbers. On the other hand, a smaller number of teachers use the drama technique and the activity-based approach. A smaller number of teachers use the drama technique and activity-based approach. For instance, T9 said students represent numbers and place their divisors in pockets; when they have more than two divisors and no pockets left, they realize the number isn't prime. T6 stated they conduct activities using the Sieve of Eratosthenes to find prime numbers.

## Discussion and conclusion

In this study, middle school mathematics teachers' PCK regarding prime numbers was examined in terms of subject matter knowledge, knowledge of students' understanding, and knowledge of instructional strategies.

In terms of subject matter knowledge, it was found that teachers' definitions of prime numbers did not consider the concept of positive divisors, ignored negative numbers, and included much broader sets of numbers. This result supports findings from various studies in the literature. In one such study, Ercire (2022) revealed that prospective teachers did not address the concept of positive divisors, the number 1, and negative numbers when defining prime numbers although they knew prime numbers, they struggled with defining them and did not pay attention to critical points. In another study, Zazkis and Liljedahl (2004) stated that prospective teachers could correctly define prime numbers but were unable to directly answer a question about the primality of the number  $151 \times 157$ . They explained that this situation stemmed from prospective teachers' incomplete understanding of the concept of prime numbers. Gürefe and Aktaş (2020), in their study with senior mathematics teacher candidates, concluded that only three out of 48 candidates were able to provide appropriate and sufficient definitions of prime numbers. Zazkis (2005) noted that prospective teachers often use negative expressions like "not divisible" or "not" when defining prime numbers, indicating they do not provide a fully accurate and complete definition of the concept. This situation supports the findings of the present study. Therefore, when defining prime numbers in textbooks and teaching practices, attention should be given to providing definitions that are appropriate for the grade level, accurate, and complete. Additionally, special course content focusing on the concept of prime numbers should be developed within teacher education programs, and activities aimed at addressing conceptual

misconceptions should be increased. The development of digital and visual materials would also provide effective support for teachers.

In their explanations regarding why 0, 1, and negative numbers are not prime numbers, teachers stated that only numbers greater than 1 can be prime, but incorrectly or incompletely identified the divisors of 0, 1, and negative numbers. This shows a lack of sufficient knowledge about divisors, leading to misconceptions about prime numbers. On the other hand, it was found that mathematics teachers' explanations regarding the importance of prime numbers or their use in daily life were not at a sufficient level, and that they tended to respond by referring to the Fundamental Theorem of Arithmetic. In addition, it was observed that the majority of mathematics teachers had limited and rule-based knowledge about how to use the Sieve of Eratosthenes. To enable mathematics teachers and prospective teachers to use the Sieve of Eratosthenes more effectively in their instruction, they can be provided with hands-on professional development opportunities. Overall, the findings show that middle school mathematics teachers' subject matter knowledge is superficial and lacks depth. Thus, instructional processes for prospective teachers and in-service professional development programs for teachers should focus on deepening their understanding. Organizing programs that enhance both conceptual knowledge and practical experience by incorporating practical sessions, utilizing concrete teaching materials, and providing personalized feedback would be beneficial.

Teachers' understanding of students' thinking forms the basis for instructional decision-making (Yurtyapan, 2018). In order for students to learn successfully, teachers need to be aware of the topics in which students may experience learning difficulties, and be able to recognize their errors and misconceptions (Fennema & Franke, 1992). The findings of the study revealed that the participating teachers had a high level of awareness regarding students' learning difficulties, errors, and misconceptions, and were able to respond accurately to the questions posed by the researchers in this regard. The teachers generally stated that students struggle to identify the divisors of numbers, believe that 1 is a prime number, think that 2 is not a prime number, and assume that all odd numbers are prime. Similarly, Akkan and Öztürk (2019) also reported that 2 is often not considered a prime number and that all odd numbers are perceived as prime. It was observed that the majority of teachers were not only able to identify students' learning difficulties, errors, and misconceptions related to the topic of prime numbers, but also able to explain their causes in a rather limited way. In sum, middle school mathematics teachers demonstrate sufficient knowledge of students' understanding and perform better in this component than in their subject matter knowledge and instructional strategies related to prime numbers. Consistent with these findings, similar results have been reported in several studies. For example, Watson and Callingham (2013) found that teachers were able to identify students' difficulties related to the concept of average; Doğruel (2019) reported that teachers were at a sufficient level in terms of both knowledge of students' understanding and subject matter knowledge; Gökkurt (2014) stated that although teachers' overall pedagogical content knowledge components were not at the desired level, their knowledge of students' understanding was relatively stronger; and Yurtyapan (2018) emphasized that the knowledge of students' understanding component was more developed compared to the other components.

When examining teachers' knowledge of instructional strategies, it was observed that they generally provided explanations based on the definition of the concept and often preferred to present rules followed by sample question solutions to address students' learning difficulties, errors, and misconceptions. It was also noted that only a limited number of teachers implemented approaches such as organizing activities, promoting group work, or providing

hints to help students recognize their own mistakes. Some teachers attributed these limitations to large class sizes, low student readiness levels, and insufficient instructional materials in schools. Reducing class sizes and supplying schools with necessary educational resources may contribute to making instructional processes more effective. Similarly, Doğruel (2019) found in his study that when eliminating students' errors in the topic of ratios and proportions, teachers generally preferred to explain concepts, state rules, and address misconceptions through verbal explanations without performing calculations. Furthermore, it was determined that teachers frequently used the presentation teaching strategy, discovery-based teaching strategy, and question-and-answer technique. However, it was also observed that strategies, methods, or techniques that actively involve students in the learning process, such as drama technique, modeling, brainstorming, and activity-based approach, were rarely employed.

It was observed that teachers often confused the concepts of strategy, method, and technique or responded using terms such as discovery-based teaching strategy, direct instruction, and drama without explicitly naming these concepts. This finding aligns with Gökbulut's (2010) study, which reported that preservice teachers confused the concepts of strategy, method, and technique. This situation reveals that teachers experience gaps and uncertainties regarding their knowledge of instructional strategies, methods, and techniques. Therefore, it can be concluded that middle school mathematics teachers' knowledge of instructional strategies related to the topic of prime numbers is insufficient. To address these shortcomings, training programs should be organized for both in-service teachers and preservice teachers, with particular emphasis on clarifying the concepts of strategy, method, and technique. Additionally, offering practical workshops and opportunities for in-class observations would be beneficial to develop pedagogical knowledge and skills.

This study was conducted with a limited sample of 12 middle school mathematics teachers. The data collection instruments measured only specific aspects of the teachers' PCK, and classroom observations were not included in the data collection process. These limitations constrain a comprehensive understanding of teachers' PCK. Future research could improve generalizability by involving a larger number of teachers from diverse geographical regions. Additionally, evaluations of PCK could be strengthened by incorporating classroom observations and student performance data. Furthermore, it is recommended that studies be conducted to assess the effectiveness of teacher education programs. Finally, research focusing on the design and evaluation of professional development programs aimed at addressing teachers' PCK deficiencies would be valuable.

## Declarations

**Acknowledgments:** The authors thank all participants for their valuable contributions.

**Funding:** This research received no external funding.

**Ethics Statements:** This research was approved by the Ethics Commission of the researcher's institution (Approval Date: 27.12.2022; Decision No: E-34659092-605.01-66908793).

This study is based on a master's thesis by the first author, supervised by the second author.

**Conflict of Interest:** The authors declare no conflicts of interest.

**Informed Consent:** Participants were informed about the study and gave written consent.

**Data availability:** Data are available from the corresponding author upon reasonable request due to confidentiality.

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