

Metacognitive Misdirection of Prospective Mathematics Teachers in Mathematical Problem Solving - an Explorative Study

Siti Mawaddah

Department of Mathematics, Universitas Negeri Malang, East Java, Indonesia
ORCID: 0000-0003-0599-6153

Purwanto Purwanto*

Department of Mathematics, Universitas Negeri Malang, East Java, Indonesia
ORCID: 0000-0003-0974-4068

I Nengah Parta

Department of Mathematics, Universitas Negeri Malang, East Java, Indonesia
ORCID: 0000-0003-4255-9323

Sisworo Sisworo

Department of Mathematics, Universitas Negeri Malang, East Java, Indonesia
ORCID: 0000-0002-1962-082X

Article history

Received:
30.12.2024

Received in revised form:
05.07.2025

Accepted:
06.08.2025

Key words:

metacognitive misdirection,
mathematical problem solving,
Polya's method, red flag

This study aims to explore the metacognitive misdirection experienced by prospective mathematics teachers during mathematical problem solving, with a focus on metacognitive skills such as planning, monitoring, and evaluating. The problem-solving framework adopted in this study is Polya's method. The research participants were selected through purposive sampling based on specific criteria: successful completion of the Calculus 1 course, the demonstrated ability to solve mathematical problems using Polya's structured problem-solving steps, indications of metacognitive misdirection, and willingness to serve as information sources. Of the sixty-three prospective mathematics teachers enrolled in the mathematics education program at Universitas Lambung Mangkurat, fourteen met the selection criteria, and three were chosen as the main participants. The data were collected through mathematical problem-solving tasks (MPST) and interviews. The MPST focused on the application of functions and derivatives. This study shows that metacognitive misdirection occurs at various steps of the problem-solving process: during the understanding-the-problem step (manifested as an error detection red flag), the carrying-out-the-plan step (characterized by error detection, lack of progress, and anomalous results red flags) and looking-back step (again marked by an error detection red flag). Among these, error detection emerged as the most frequently observed red flag. The study also identifies two distinct types of metacognitive misdirection processes: pseudo-metacognitive misdirection and essential metacognitive misdirection.

*Correspondency: purwanto.fmipa@um.ac.id

Introduction

Problem-solving is a fundamental component of mathematics education. It involves applying prior knowledge and skills to address unfamiliar situations (Ardiyaningrum et al., 2019; Carson, 2007; Haury, 2002; Krulik & Rudnick, 1988). Mathematical problem-solving is a process of interpreting a situation mathematically, typically involving several iterative cycles of expressing, testing, and revising mathematical interpretation, as well as sorting out, integrating, modifying, altering, or refining clusters of mathematical concepts from various topics within and beyond mathematics (Kuzle, 2013; Lesh & Zawojewski, 2007). Polya (1985) proposed four steps in problem-solving: understanding the problem, devising a plan, carrying out the plan, and looking back. Prospective mathematics teachers require strong problem-solving skills not only to solve problems themselves but also to guide their students in developing similar abilities.

Problem-solving is closely related to metacognition. Several experts have offered definitions of metacognition, commonly described as “thinking about thinking” (Aljaberi & Gheith, 2015; Flavell, 1979; Kuzle, 2013; Lai, 2011; Livingston, 2003; Papeleontiou-Louca, 2003; Schoenfeld, 1985). Flavell defined metacognition as knowledge concerning one’s cognitive processes and products or anything related to them (Ozdogan et al., 2019).

When solving mathematical problems, metacognitive skills are essential. These include planning, monitoring, and evaluating. Metacognitive skills enable individuals to regulate and supervise their learning or problem-solving processes. They encompass the ability to plan, monitor, control, and evaluate cognitive activities, which are central to cognitive regulation (Ader, 2019; Whitebread et al., 2009). Planning involves selecting appropriate strategies for a task. Monitoring refers to a person's awareness of their performance while engaged in the task. Control and evaluation involve reviewing and assessing the entire process (Schraw & Moshman, 1995). Metacognitive skills enhance students’ ability in regulating their thinking processes and improving thinking skills in problem-solving (Knox, 2017; Joseph, 2010; Schraw & Graham, 2010).

In problem-solving, the metacognitive process does not always succeed; sometimes there is a failure in the metacognitive process. According to Goos (2002), there are three metacognitive failures: metacognitive mirages, metacognitive blindness, and metacognitive vandalism. Metacognitive failures in mathematical problem-solving have been widely studied (Faradiba, et al., 2019; Faradiba & Alifiani, 2020; Goos, 2002; Huda et al., 2018, 2019; Kaya & Kepceoglu, 2022; Rozak et al., 2018). Most of these studies focused on prospective teachers (Faradiba, et al., 2019; Faradiba & Alifiani, 2020; Huda et al., 2018, 2019; Kaya & Kepceoglu, 2022; Rozak et al., 2018). Rozak et al., (2018), for example, described the process of identifying students' metacognitive failures in solving mathematical problems. The results of their study showed that each subject studied experienced metacognitive failures of different types. Huda et al., (2019) explored students' metacognitive failures in solving mathematical problems based on their metacognitive activities. Huda et al., (2018) were conducted to reveal metacognitive errors in the evaluation of students' metacognitive failures in solving mathematical problems. Faradiba et al., (2019) investigated how metacognitive failures occurred during problem-solving experienced by prospective teachers about mathematical anxiety. Kaya & Kepceoglu (2022) identified eight different metacognitive failure behaviors were found, consisting of metacognitive mirage twice, metacognitive blindness three times, and metacognitive vandalism three times. However, these studies mainly focus on the metacognitive failures as classified into blindness, vandalism, or mirage. Stillman (2011) added two other types of metacognitive failures: metacognitive misdirection and

metacognitive impasses.

The limitations of previous research, as discussed above, are further supported by findings from a preliminary study conducted at Universitas Lambung Mangkurat, Indonesia. This preliminary study focused on prospective mathematics teachers in mathematical problem-solving. When working on the mathematical problem task, prospective mathematics teachers are asked to carry out metacognition according to the instructions given on the mathematical problem task sheet. The results showed that 89% of the 64 prospective mathematics teachers failed to solve mathematical problem tasks, and 22% demonstrated indications of metacognitive misdirection.

Metacognitive misdirection occurs when a red flag is recognized and an appropriate response is initiated, but the problem-solving goal is still not achieved. A red flag indicates difficulties, process errors, or errors resulting from attempts at problem solving. It serves as a warning to pause, step back, and immediately take appropriate action against potential failures in problem-solving (Goos, 2002; Goos et al., 2000). A red flag acts as a trigger for metacognitive activity when someone becomes aware of certain difficulties (Stillman, 2011). Red flag situations in problem-solving occur when someone experiences a lack of progress, error detection, or anomalous results.

Lack of progress (LP)—the first type of red flag—should prompt someone to revisit the problem analysis process to reevaluate the suitability of the selected strategy and determine whether to continue, keep the helpful information, or give up completely. In the latter scenario, someone will probably need to reevaluate how well they comprehend the issue and look for new information or an alternative strategy. Error detection (ED), the second red flag, ought to cause the computations done thus far to be checked and corrected. Anomalous results (AR), the third red flag, should cause a calculation check (evaluate strategy execution), followed if required by reconsidering the strategy if attempts to verify the solution show that the answer does not satisfy the problem requirements or does not make sense. Figure 1 illustrates metacognitive misdirection, modified from the scenarios of metacognitive failure by Goos (2002) and Stillman (2012).

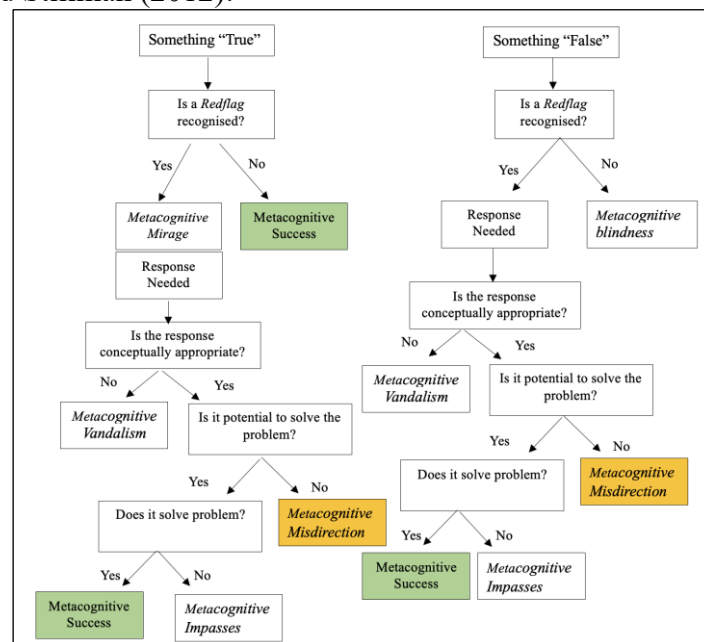


Figure 1. Metacognitive Misdirection

Based on Figure 1, metacognitive misdirection can occur either from something that is initially true or false. This condition occurs when a red flag (LP/ED/AR) is recognized and responded to in a way that seems appropriate but actually does not help solve the problem or does not lead to achieving the goal of solving the problem. Metacognitive misdirection is the central focus of this study.

This study aims to explore the metacognitive misdirection of prospective mathematics teachers in mathematical problem-solving. Findings from a preliminary study revealed types of metacognitive failure beyond those identified by previous researchers - specifically metacognitive misdirection. Furthermore, it was observed that these prospective teachers exhibited different characteristics of metacognitive misdirection. Stillman (2011) successfully identified a type of metacognitive failure other than that proposed by Goos (2002), namely metacognitive misdirection. However, his research has not explored in more depth how the metacognitive misdirection process occurs in the problem-solving process. Indeed, investigating metacognitive misdirection in mathematical problem-solving remains a pressing and exciting area of research today. This research offers valuable insights for both prospective teachers and lecturers. For prospective teachers, understanding metacognitive misdirection may help them anticipate and overcome it, supporting their development as more effective problem solvers. For lecturers, the findings can inform the design of learning activities that promote metacognitive success among their students.

Methodology

Research Design

This study employed an exploratory qualitative approach to examine the metacognitive misdirection of prospective mathematics teachers in mathematical problem-solving. This research focused on the metacognitive misdirection in relation to metacognitive skills namely planning, monitoring, and evaluating. The problem-solving framework used in this study is Polya's problem-solving framework, which included the steps of understanding the problem, devising a plan, carrying out the plan, and looking back.

Research Participants

This research involved 63 prospective mathematics teachers in the mathematics education program at Universitas Lambung Mangkurat. The participant selection used a purposive sampling technique. The participants were selected based on several criteria: they had passed the Calculus 1 course (because the test material on mathematical problem solving covered the application of functions and derivatives), were able to solve mathematical problems using Polya's systematic problem-solving steps (understanding, planning, carrying out, and looking back), showed indications of metacognitive misdirection, and were willing to provide. Based on the participants selection criteria, out of 63 prospective mathematics teachers, 14 prospective mathematics teachers met the criteria. However, out of 14 prospective mathematics teachers who met the criteria, only four prospective mathematics teachers showed consistent metacognitive misdirection after being given tasks 1 and 2, and two of them experienced similar metacognitive misdirection processes. As a result, the main participants in this study were three prospective mathematics teachers.

Data collection

Data collection techniques in this research were tests and interviews. The research instruments are mathematical problem-solving tasks (MPST) and interview guides. The MPST content is related to the application of functions and derivatives studied by prospective mathematics teachers in the Calculus 1 course. In addition, the MPST requires planning, monitoring, and evaluating skills in its completion, allowing participants to experience red flags at the step of understanding the problem, devising a plan, carrying out the plan, or looking back, and the MPST given the potential to give rise to metacognitive misdirection. MPST was given twice including MPST 1 and MPST 2.

The first, the MPST 1, was given to 63 prospective mathematics teachers. Based on the test results, 14 prospective mathematics teachers met the criteria for having passed the Calculus 1 course, were able to solve mathematical problems using Polya's systematic problem-solving steps, showed indications of metacognitive misdirection, and were willing to provide. These 14 prospective mathematics teachers were then given the MPST 2 with the think-aloud technique. Think-aloud technique is done by asking participants to say whatever comes to their mind while completing the MPST 2. This may include what they see, think, do, and feel. The think-aloud for 14 prospective mathematics teachers is done alternately, with different schedules from each other. Then interviews were conducted to obtain more in-depth information about the metacognitive misdirection experienced by these prospective mathematics teachers in solving MPST. Interview and test activities using the think-aloud technique were recorded audio-visually. Of the 14 prospective teachers who worked on MPST 2 using the think aloud technique and were then interviewed, four prospective mathematics teachers consistently showed experiencing metacognitive misdirection, and two of the four prospective mathematics teachers experienced a similar metacognitive misdirection process. Thus, the data presented in this study are data from three prospective mathematics teachers. The MPST given during the research has gone through a logical and empirical validation process. MPST 2 can be seen in Figure 2.

MASALAH TARIF KAMAR HOTEL
Pak Ahmad mendapatkan warisan sebuah hotel dari orang tuanya. Dia ingin mengembangkan usaha hotelnya, tetapi dia tidak memiliki pengalaman yang cukup dalam manajemen hotel. Hotel ini memiliki 50 kamar. Pak Ahmad mendapatkan informasi sebelumnya bahwa jika tarif normal Rp.200.000,00/hari untuk setiap kamar, maka tingkat hunian adalah 100%. Dia juga diberitahu bahwa setiap kenaikan 5% dari tarif normal, maka tingkat hunian akan berkurang 2%. Biaya layanan dan pemeliharaan untuk setiap kamar yang dihuni adalah Rp.40.000,00/hari. Pak Ahmad ingin membuat tarif baru sehingga dapat memaksimalkan keuntungannya. Bantulah Pak Ahmad untuk menghitung berapa tarif baru perhari yang harus dikenakan per kamar untuk memaksimalkan keuntungannya dan berapa keuntungan yang diperoleh Pak Ahmad pada tarif baru tersebut! Adaptasi dari: https://engineering.purdue.edu/ENE/Research/SGMM/Problems/CASESTUDIESKIDSWEB/hotel.s.htm
HOTEL ROOM RATES PROBLEM
Mr. Ahmad inherited a hotel from his parents. He wants to expand his hotel business, but he does not have enough experience in hotel management. This hotel has 50 rooms. Mr. Ahmad received previous information that if the normal rate is Rp. 200,000.00/day for each room, then the occupancy rate is 100%. He was also informed that for every 5% increase from the normal rate, the occupancy rate will decrease by 2%. The service and maintenance costs for each occupied room are Rp. 40,000.00/day. Mr. Ahmad wants to create a new rate so that he can maximize his profit. Help Mr. Ahmad calculate how much the new daily rate should be charged per room to maximize his profit and how much profit Mr. Ahmad gets from the new rate! Adapted from: https://engineering.purdue.edu/ENE/Research/SGMM/Problems/CASESTUDIESKIDSWEB/hotels.htm

Figure 2. Mathematical Problem-Solving Task (MPST)

Table 1 describes the components of metacognitive skills that appear at each step of Polya's problem-solving.

Table 1. Metacognitive Skills Components

Metacognitive Skills	Description	Coding
Understanding the Problem Step		
Planning	● Thinking about why to calculate in advance the increase in hotel room rates.	PU-1
	● Thinking about why to calculate the reduction in the number of occupied rooms if there is an increase in rates.	PU-2
	● Thinking about the reasons why service and maintenance costs are costs incurred by the hotel.	PU-3
Monitoring	● Retracing the flow of thinking about the information known and asked.	MU-1
	● Thinking about why to make a relationship between the new rate and the number of rooms to be occupied.	MU-2
Evaluating	● Thinking about why thinking that the information given in the problem is sufficient or not to achieve the goal of the problem.	EU-1
	● Revising information about what is known and what is asked (if necessary).	EU-2
Devising Plan Step		
Planning	● Relating the given problem to concepts or knowledge that have been learned previously.	PD-1
	● Thinking about the reasons for choosing a pattern-finding strategy, namely by making a table/making a mathematical model then solving it by determining the maximum value of a quadratic function/making a mathematical model, then solving it using the concept of derivatives.	PD-2
Monitoring	● Re-checking the rules/relationships that have been compiled based on the information that is known or asked.	MD-1
Evaluating	● Thinking about the reasons that the chosen strategy is appropriate or not used to achieve the problem's objectives.	ED-1
	● Changing the strategy plan used to solve the problem (if necessary).	ED-2
Carrying Out the Plan Step		
Planning	● Thinking back about the components that must be included in the table/example that has been compiled.	PC-1
	● Thinking back to the steps that must be taken next to solve the problem.	PC-2
Monitoring	● Retracing the sequence of work steps using the strategy that has been implemented.	MC-1
	● Identifying difficulties, errors, or oddities in each step of solving the problem is accompanied by specific reasons.	MC-2
Evaluating	● Rethinking the process/calculation results obtained in the table that has been prepared.	EC-1
	● Revising the process/calculation results from the problem-solving obtained (if necessary).	EC-2
Looking Back Step		
Planning	● Thinking of a plan on how to re-examine the process and results of problem-solving.	PL-1
Monitoring	● Retracing the process, results, and conclusions obtained from the problem-solving process.	ML-1
Evaluating	● Rethinking the conclusions obtained whether they are by the objectives of the problem.	EL-1
	● Revising the conclusions obtained in the solution (if necessary).	EL-2

Data analysis

In this research study, MPST 2 results, think-aloud recordings, and interviews were analyzed using a three-step procedure: data summarizing, presentation, and conclusion drawing. During the summarization step, the researcher highlighted key information by condensing lengthy responses into more concise forms. The summarized data included written responses and think-aloud recordings from MPST 2. To confirm accuracy, this data was then cross-referenced with interview data. Voice recordings and interview notes were transcribed and included in the data summary.

The information was chosen and methodically arranged by the researcher after data collection, and it was then presented as a narrative. The metacognitive misdirection of prospective mathematics teachers was also shown using a flow diagram to enhance understanding of fundamental differences in their metacognitive misdirection across the problem-solving steps: understanding the problem, devising a plan, carrying out the plan, and looking back. Concluding the data collected from participants is the final step. To strengthen the validity of the findings, triangulation was conducted by comparing the researcher's interpretations with the participants' MPST responses, think-aloud data, and interview transcripts. In this study, all qualitative data collected were analyzed descriptively.

Results and Discussion

Results

This section presents a descriptive overview of the results of data analysis regarding metacognitive misdirection experienced by prospective mathematics teacher students, S1, S2, and S3, when completing MPST 2. The metacognitive misdirection experienced by S1, S2, and S3 occurred at different problem-solving stages: S1 at the looking back step, S2 at the carrying out the plan step, and S3 at the understanding the problem step. Planning, monitoring, and evaluating are the components of metacognitive skills observed in this study. The metacognitive misdirection processes of each subject are visualized in Figure 4, Figure 6, and Figure 8.

The completion of the MPST by S1, S2, and S3 showed a complete Polya's problem-solving steps: understanding the problem, devising a plan, carrying out the plan, and looking back on the problem-solving results of the given MPST.

Description of Metacognitive Misdirection of Subject 1 (S1)

S1 experienced metacognitive misdirection at the final step of problem-solving, specifically during the looking back step. Throughout the problem-solving process, S1 encountered two key red flags: LP and ED. At the step of understanding the problem step, no red flags were observed; S1 demonstrated a clear understanding of the problem context. LP occurs at the step of devising a plan, when S1 plans a strategy using a table to solve the problem, but S1 showed a lack of progress at this step because after S1 tries several possible new rates, S1 feels that with a strategy using a table, he must calculate manually while the time available is limited. ED occurred during both the carrying out the plan and looking back step. In the carrying out the plan step, S1 made a miscalculation in estimating profit when the tariff was increased by 100%. In the final step, ED reappeared when S1 drew an incorrect conclusion about the new tariff required to achieve maximum profit.

At the looking back step, S1 reexamined his initial, incomplete conclusion. Initially, he stated only the percentage increase in tariff and the maximum profit. He later revised this conclusion to include the required new tariff to achieve maximum profit (ML-1, EL-1, EL-2). However, the revised conclusion still led to an incorrect answer: S1 stated the new tariff as Rp.330,000, whereas the correct amount was Rp.370,000. The conclusion made by S1 is shown in Figure 3.

Jadi, tarif baru perhari yang harus dikenakan per kamar untuk memaksimalkan keuntungan adalah 85% dari tarif normal atau sebesar Rp 330.000,- per hari untuk setiap kamar, dengan keuntungan sebesar Rp 10.890.000,-.	So, the new daily rate that must be charged per room to maximize profits is 85% of the normal rate, or Rp. 330,000 per day for each room, with a profit of Rp. 10,890,000.
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Figure 3 Conclusions made by S1 on MPST

S1 recognized and appropriately responded to red flags at the devising a plan and carrying out the plan steps, remaining on the correct solution path. However, at the looking back step, S1 experienced error detection (ED), as shown in the interview results in Table 2.

Table 2. Transcript of Interview Between P and S1 that Shows S1 is Aware of Red Flag Error Detection

Transcript of Interview	
P	: "Are you sure of your conclusion?"
S1	: "I should be sure, ma'am (her tone of voice sounded hesitant, then looked back at the scribble sheet). I should be sure, ma'am (looking back at the scribbles); for the profit answer there should be no mistake ma'am, but in the profit section, the profit is correct at 85% of the normal rate but the amount seems wrong. I checked in the scribbles on the list earlier; the calculation is correct. " (ED)
P	: "At the conclusion, did you rethink your conclusion?"
S1	: "For the conclusion, not really; the conclusion is only once ma'am, the conclusion is based on the results of the previous work, and obtained that $a = 17$, so only once, ma'am."

Based on the interview results presented in Table 2, S1 revealed that he sensed an error in his answer regarding the new tariff to be charged and he attempted to identify the error by reviewing the calculation scribble sheet (ML-1) but was unable to locate it. This indicates that S1 experienced metacognitive misdirection. Metacognitive misdirection occurs when a red flag is recognized and the response seems appropriate, yet the response fails to achieve the intended problem-solving goal. The metacognitive misdirection process experienced by S1 can be seen in Figure 4.

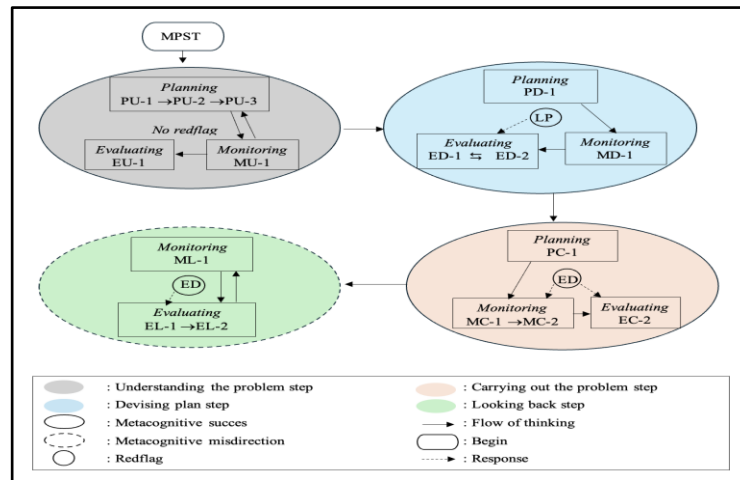


Figure 4. Metacognitive Misdirection of S1

Based on Figure 4, S1 demonstrated metacognitive skills such as planning, monitoring, and evaluating throughout the problem-solving process. However, S1 experienced metacognitive misdirection at the looking back step (with the types of red flag ED) due to forgetting the rule he had formulated during the carrying out the plan step. The rule formulated by S1 is the profit obtained by multiplying the net income per room (already reduced by service and maintenance costs) by the number of rooms occupied. Meanwhile, what is asked in the MPST is the new rate that must be charged, this means that the new rate must also be added to the service and maintenance costs per room. S1 understands the information according to the context of the problem given, as shown in the MPST response and interview, indicating awareness that the stated rates already include service and maintenance costs. S1 successfully constructed a mathematical model and demonstrated strong procedural skills. However, due to a lack of careful review of the rule he created, the final answer—particularly regarding the new rate—was inaccurate.

Description of Metacognitive Misdirection of Subject 2 (S2)

Similar to S1, S2 did not encounter any red flags at the understanding problem step, and the information provided was interpreted appropriately within the context of the problem given. Metacognitive misdirection of S2 occurred at the step of carrying out the plan. During problem-solving, S2 encountered two key red flags: ED and LP. LP occurs at the step of devising a plan and the step of carrying out the plan. ED occurs at the step of carrying out the plan. S2 successfully recognized the LP red flag at the devising a plan step, where he struggled to convert the MPST problem into a mathematical model. In response, S2 appropriately shifted from using a mathematical model to constructing a table (ED-2), thereby remaining on the correct solution path.

At the carrying out the plan step, S2 again recognized both LP and ED red flags. ED encountered when S2 made a calculation error, assuming that a 50% increase in the tariff would result in 30 rooms being occupied. S2 then calculated the profit for this scenario, which amounted to Rp7,800,000. Subsequently, S2 re-estimated that maximum profit would be achieved with a tariff increase of approximately 35%, 40%, or 45%. When determining that 42 rooms would be occupied with a 40% increase, S2 retraced each step of the solution (MC-1) and re-evaluated the calculation results (EC-1).

Through this review, S2 realized he had made a calculation error when determining the

number of rooms occupied when the tariff was increased by 50% from the normal tariff, which was originally 30 rooms that should be 40 rooms (MC-2). S2 responded appropriately by correcting the error (EC-2). This process is further illustrated by the think-aloud transcript presented in Table 3.

Table 3 Transcript of Think-Aloud that Shows S2 Realizing Calculation Error and Correcting it.

Transcript of Think-Aloud

"To maximize the profit, if it is 50%, the profit is Rp.7,800,000. That means it's even less if it's 50%. This means it's too much. We'll try again at 30%. A 30% increase means the tariff is 260,000 for 44 rooms, so it's 11 million, higher than the previous one, but with maintenance costs, maintenance costs remain at 40,000 per day (counting), which means the profit is Rp.9,680,00.00. Okay, this means that the profit is still higher than the usual tariff, which is 8 million. This means that the profit is around 40%, 45% or 35%. If 40% means the rate is 380,000 per day, it means 42 rooms.

This should be 40 rooms (while pointing to the answer sheet that shows the calculation results when the tariff is increased by 30%), **is the profit wrong? This should be 40, not 30 rooms, it means there was an error here, that's why it decreased so much, where was it (calculating)? Oh, the occupancy rate decreased by 2%, so 20%. Uh wait, that means there was a mistake; the maintenance fee is 40,000** (correcting the wrong calculation)."

After finding and correcting the mistake, S2 thought that with a 50% tariff increase, the profit would still increase. S2 decided to continue his calculation by calculating the profit obtained if the tariff is increased by 100%. S2 found that the profit obtained if the tariff is increased by 100% is Rp.10,800,000. After that, S2 had wanted to change the strategy used, namely by trying to make a mathematical model of the MPST given, but S2 had difficulty in compiling the mathematical model. In this case, S2 experienced red flag LP. S2 then gave an appropriate response to the red flag he experienced by deciding to return to using the initial strategy he used, namely the strategy of making tables. The problem-solving results obtained by S2 at the end of carrying out the plan step can be seen in Figure 5.

30%	260.000	44 kamar	Rp. 9.680.000	100 - 1
40%	280.000	42		60
50%	380.000	32 kamar	Rp. 10.800.000	70
60%	400.000 hari	30 kamar		80
150%	500.000	20 kamar	Rp. 5.200.000	
130%	480.000	24 kamar	Rp. 10.080.000	
125%	450.000	25 kamar	Rp. 10.250.000	
		28 kamar	Rp. 11.420.000	

Figure 5. Results of Problem-Solving by S2 at the Step of Carrying out the Plan

At the end of the step of carrying out the plan, S2 obtained the calculation results that the maximum profit will be obtained if the tariff is increased by 100%, namely with a new tariff of Rp.400,000.00 and the maximum profit is Rp.10,800,000.00. The problem-solving results obtained by S2 at the step of carrying out the plan are still not correct. Thus, S2 experienced metacognitive misdirection at the step of carrying out the plan with ED and LP red flags. In this case, S2 realized that he was in a state of no progress (when changing the strategy by compiling a mathematical model of the MPST) and S2 also realized that he made a mistake (when determining the number of rooms occupied if the tariff was increased by 50% and 105%). S2 tried to give an appropriate response by re-using the table strategy and re-examining the results of his calculations but the problem-solving results obtained by S2 were still not correct. The complete metacognitive misdirection process experienced by S2 can be seen in Figure 6.

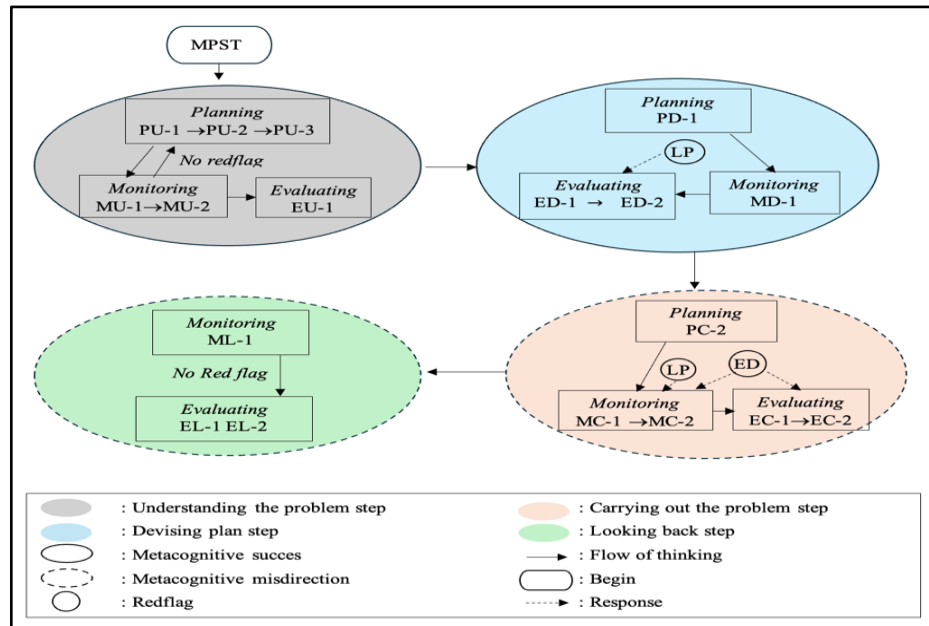


Figure 6. Metacognitive Misdirection of S2

Based on Figure 6, during problem-solving process, S2 demonstrates planning, monitoring, and evaluating. S2 experienced metacognitive misdirection at the carrying out the plan step (with the types of red flag: LP and ED). This occurred because, during the carrying out the plan step, S2 used a tabular strategy to calculate the hotel's maximum profit by selecting specific percentage increases in room rates. However, he only calculated the profit for limited percentages (e.g., 5%, 10%, 15%, 35%, 40%, 45%, 50%, 100%, 105%, 110%, 125%, 130%, and 150%), missing key intervals that could have yielded the maximum profit. Although S2 demonstrated an understanding of the given information, selected an appropriate strategy, and showed procedural skills, the omission of several crucial percentage values led to an inaccurate final answer.

Description of Metacognitive Misdirection of Subject 3 (S3)

S3 experiences metacognitive misdirection during the understanding of the problem step, triggered by ED. He also encountered ED, LP and AR at the carrying out the plan step. In the understanding problem step, S3 incorrectly assumed that service and maintenance costs were expenses borne by hotel guests. As shown in Figure 7, when calculating daily revenue, he added these costs, resulting in a misinterpretation of the problem context.

Diketahui : - Ada 50 kamar - Tarif normal Rp 200.000/hari → tingkat hunian 100% - Kenaikan 5% tarif normal → tingkat hunian berkurang 2% - Biaya layanan dan pemeliharaan setiap kamar → Rp 40.000/hari Dit: keuntungan maksimum Penyelesaian : # Tarif Normal Hasil diperoleh per hari → $\begin{array}{r} \text{Rp } 200.000 \times 50 = 10.000.000 \\ \text{Rp } 40.000 \times 50 = 2.000.000 \\ \hline \text{Rp } 12.000.000 \end{array}$ # Model Setiap kenaikan bertambah Rp 10.000/hari kamar berkurang huniannya	Known: - There are 50 rooms. - The normal rate is Rp. 200,000.00/day → the occupancy rate is 100%. - Every 5% increase from the normal rate → the occupancy rate will decrease by 2%. - The service and maintenance costs for each occupied room are Rp. 40,000.00/day. Asked : Maximum profit # Normal rate Income received perday → $\begin{array}{r} \text{Rp.}200.000 \times 50 = \text{Rp.}10,000,000 \\ \text{Rp.}40.000 \times 50 = \text{Rp. } 2,000,000 \\ \hline \text{Rp.}12,000,000 \end{array}$ # Modelling For every increase from the normal rate, an increase of Rp. 10,000 is added. Reduces 1 room for the occupancy.
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Figure 7. S3 Answer at the Step of Understanding the Problem

This is reinforced by the interview results shown in Table 4.

Table 4 Transcript of Interview between P and S3 that Shows S3 Thinking about Information regarding Service and Maintenance Costs

Transcript of Interview		
P	:	"Did you think about service and maintenance costs?"
S3	:	"The service and maintenance fee had a thought, but I thought it was per day. It turns out that when I read it for each room occupied, it is 40,000 per day, which means that 1 room is IDR 40,000 so it is added like this. I missed it at first."
P	:	"Did you think or not that Rp40,000 was issued by the hotel or hotel guests?"
S3	:	"Because usually the cost of maintenance services is from other parties, from earlier, we immediately think of the consumer."
P	:	"Why do you assume that service and maintenance costs are incurred by hotel guests?"
S3	:	"If, for example, we need anything, the important thing is that we call what service is called and then if we want to need services in the room like that, maybe there are extras, or cleaning, and all kinds of things."

Based on Table 4, S3 experienced an ED while interpreting the information about service and maintenance costs. Initially, he assumed that the Rp40,000.00 fee was the total cost per day for all rooms. In response, he retraced his thoughts (MU-1) and correctly recognized that the fee was charged daily per room. Although this was an appropriate response to the initial red flag, S3 mistakenly believed that the service and maintenance costs were borne by hotel guests. He maintained this incorrect assumption throughout the problem-solving process, resulting in an inaccurate final answer. Thus, although the ED red flag was addressed, the resolution did not align with the problem's objectives, indicating metacognitive misdirection at the understanding of the problem step. The complete misdirection process is illustrated in Figure 8.

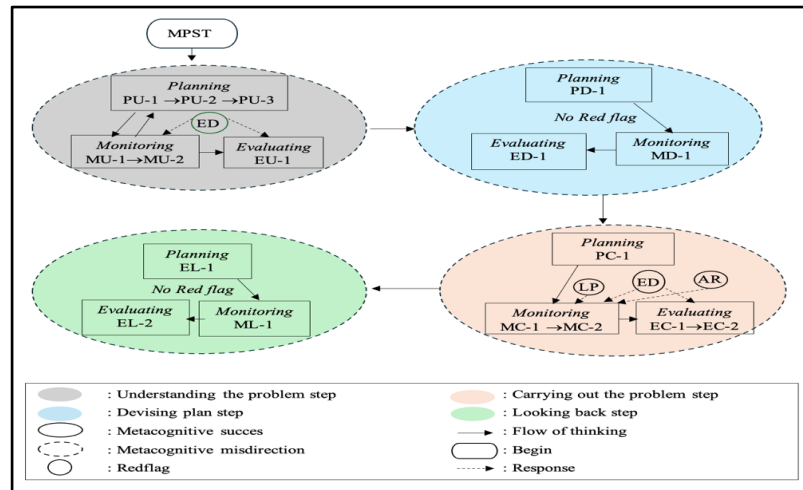


Figure 8. Metacognitive Misdirection of S3

Based on Figure 8, S3 experienced metacognitive misdirection during the understanding of the problem step, triggered by an ED red flag. At the planning step, S3 did not encounter any red flags and successfully selected a strategy that combined mathematical modeling with tabular representation. However, during the carrying out of the plan step, S3 faced multiple red flags (LP, ED, and AR). Although he recognized these issues and attempted to respond appropriately using planning, monitoring, and evaluating his final solution remained incorrect. This misdirection stemmed from a fundamental misunderstanding at the initial step, where S3 assumed that the service and maintenance costs in the MPST were expenses paid by hotel guests. This misconception persisted in the carrying out the plan, affecting all subsequent decisions, despite his efforts to follow the chosen strategy. In the looking back step, no red flags were observed. S3's conclusions aligned with his initial understanding and applied strategy. However, because that understanding was flawed from the beginning, the overall problem-solving outcome was incorrect.

This study revealed important aspects of the metacognitive misdirection processes experienced by prospective mathematics teachers during mathematical problem-solving. One key finding concerns the types of red flags associated with instances of metacognitive misdirection, which were predominantly error detection (ED) red flags. These findings are summarized in Table 5.

Table 5 Types of Red Flags Observed during Metacognitive Misdirection

Polya's Problem-solving Step	Type of Redflag	Participant
Understanding the problem	ED	S3
Devising a plan	-	-
Carrying out the plan	ED, LP, AR	S2, S3
Looking back	ED	S1

Another key finding in this study relates to the types of metacognitive misdirection processes, which include **pseudo-metacognitive misdirection** and **essential metacognitive misdirection**. **Pseudo-metacognitive misdirection** occurs when the initial understanding aligns with the context of the problem and is reconstructed toward the intended problem goals, but the process is derailed by minor errors (e.g., calculation inaccuracies), resulting in an incorrect solution. This type was experienced by participants S1 and S2. In contrast, **essential metacognitive misdirection** begins with a fundamental misunderstanding of the problem context. Although the reconstruction process follows a logical path based on that

misunderstanding, it ultimately leads to an incorrect solution. This type was observed in S3.

Discussion

This study found several important aspects about the metacognitive misdirection in prospective mathematics teachers during mathematical problem-solving. The findings highlight **variations in the steps at which misdirection occurred**. S1 experienced metacognitive misdirection at the **looking back** step, marked by an **ED** red flag. S2's misdirection occurred during the **carrying out the plan** step, involving **ED** and **LP** red flags. Meanwhile, S3 experienced misdirection at the **understanding of the problem** step, also marked by an **ED** red flag. As a form of **metacognitive failure**, metacognitive misdirection is proposed as a complementary category alongside previously identified types such as **metacognitive blindness**, **metacognitive mirage**, and **metacognitive vandalism**.

These findings of this study are consistent with Goos (2002), who also found that **metacognitive failures can occur at various steps of problem-solving**. However, unlike Goos's study, which identified **metacognitive blindness**, **mirage**, and **vandalism**, the present study focused on a different type—**metacognitive misdirection**. Additionally, the problem-solving framework used differs. Goos (2002) employed the Artz and Armour-Thomas (1992) framework, in which failures were observed at the **exploration**, **implementation**, and **verification** step. According to Ozdogan et al., (2019), the exploration and implementation steps align with the **carrying out the plan** step in Polya's framework, while the verification step corresponds to **looking back**. Notably, this study extends prior findings by identifying that **metacognitive misdirection can also occur during the step of understanding the problem**, which was not observed in Goos's framework.

Another important finding in this study is that the most dominant red flag associated with metacognitive misdirection is ED, which was observed in all participants. This suggests that, during problem solving, individuals often recognize internal signals indicating an error has occurred. However, these signals are frequently ignored or followed by responses that fail to align with the intended goals of the problem. Such outcomes are often linked to underdeveloped metacognitive processes, particularly **evaluation**, which is not exercised optimally. In some cases, **overconfidence** also contributes to this misdirection, causing individuals to persist with incorrect understandings or conclusions. This shows that metacognitive skills have an important role in overcoming the metacognitive misdirection that occurs. Based on the results of this study, although participants were aware of inconsistencies or errors in their problem-solving processes or outcomes, failure still occurred due to their inability to respond effectively. This finding is consistent with Stillman's (2020) view that individuals experiencing metacognitive misdirection often struggle to act upon the errors they detect.

The final finding of this study relates to the types of metacognitive misdirection processes, which include **pseudo-metacognitive misdirection** and **essential metacognitive misdirection**. Pseudo-metacognitive misdirection, as experienced by S1 and S2, is not fundamental, as individuals could likely recognize and correct their mistakes if given additional time or opportunities for reflection. Providing extended time for reflection during the problem-solving process may help individuals overcome metacognitive misdirection by enhancing their awareness and regulation of cognitive processes, ultimately leading to successful problem solving (Becker et al., 2023; Huda & Marzal, 2023; Reinhard et al., 2021; Winarti et al., 2022). In contrast, essential metacognitive misdirection is deeper and more

systemic. It occurs when an individual fails to recognize an error due to a fundamental flaw in their thinking process, particularly stemming from a mistaken belief about their understanding of the problem. In other words, this type of misdirection arises from internal misconceptions or overconfidence, making it difficult to correct through simple feedback. Instead, it requires deeper interventions, such as intensive metacognitive guidance. Providing such guidance has been shown to significantly improve metacognitive abilities, which in turn enhances students' mathematical problem-solving skills (Habib et al., 2024; Özsoy & Ataman, 2009).

Previous studies by Aşık & Erkin (2019); Güner & Erbay (2021); Krieger et al., (2022); Ozdogan et al., (2019); Tachie (2019) show the importance of metacognition in mathematics problem-solving, both in terms of monitoring the thinking process and error detection. The research findings on metacognitive misdirection strengthen these results, especially by paying attention to the types of red flags that occur in prospective mathematics teachers, such as error detection, lack of progress, and anomalous results, as well as variations in metacognitive misdirection that occur at the steps of mathematical problem-solving. This study also shows that metacognitive misdirection can occur due to errors, whether the understanding of the information in the problem is correct or not. This understanding is important for prospective mathematics teachers, as they need to understand how to improve the results of their thinking process and detect errors that may occur at various steps of problem-solving. This research makes an important contribution in adding insight into how metacognitive misdirection can be addressed by improving metacognitive skills for prospective mathematics teachers. Teachers and curriculum designers can use the findings to improve metacognitive training in mathematics education. Furthermore, further research is needed to develop intervention strategies for addressing both pseudo-metacognitive misdirection and essential metacognitive misdirection that arise during mathematical problem-solving. It is also important to design metacognitive learning models that actively engage planning, monitoring, and evaluating skills, thereby enabling prospective teachers to respond appropriately to error detection and succeed in metacognitive processes during problem-solving.

Conclusion

This research focuses on the metacognitive misdirection of prospective mathematics teachers in mathematical problems, in terms of metacognitive skills (i.e., planning, monitoring, and evaluating). Metacognitive misdirection, as a type of metacognitive failure, is determined based on the response to red flags. Metacognitive misdirection occurs when a red flag (LP/ED/AR) is recognized, then the red flag gets an appropriate response but does not achieve the expected problem goal. Previous research reported metacognitive failure (i.e., metacognitive blindness, metacognitive vandalism, or metacognitive mirage). Based on a preliminary study, it was also found that there were types of metacognitive failure other than those studied by previous researchers, namely metacognitive misdirection, but it was found that there were different characteristics of metacognitive misdirection experienced by these prospective teachers. To these conditions, a more specific study is needed.

This research shows that metacognitive misdirection occurs at various steps of the problem-solving process: during the **understanding** problem step (ED red flag), **carrying out the plan** step (ED, LP, and AR red flags), and **looking back** step (ED red flag). Among these, **error detection (ED)** is the most frequently observed red flag. The study also identifies two types of metacognitive misdirection processes: **pseudo-metacognitive misdirection**, and **essential metacognitive misdirection**. **Pseudo-metacognitive misdirection** occurs when the initial understanding aligns with the context of the problem and is reconstructed toward

the intended problem goals, but the process is derailed by minor errors (e.g., calculation inaccuracies), resulting in an incorrect solution. In contrast, **essential metacognitive misdirection** begins with a fundamental misunderstanding of the problem context. Although the reconstruction process follows a logical path based on that misunderstanding, it ultimately leads to an incorrect solution.

Declarations

Acknowledgments: This paper is derived from the doctoral dissertation of the first author, under the supervision of the second, third, and fourth authors.

Funding: This research was supported by the Beasiswa Penyelesaian Studi (BPI) scholarship from the Lembaga Pengelola Dana Pendidikan (LPDP), Ministry of Finance of the Republic of Indonesia.

Ethics Statements: The study was conducted in accordance with the strictest ethical guidelines, according to the authors.

Conflict of Interest: The authors declare that there is no conflict of interest regarding the publication of this paper.

Informed Consent: Informed consent was obtained from all participants involved in the study. Participants were informed about the purpose of the research, procedures, and their right to withdraw at any time without penalty.

Data availability: The data that support the findings of this study are available from the corresponding author upon reasonable request.

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