Id: 1538140

Participatory Educational Research (PER)
Vol.12(3), pp. 22-50, May 2025
Available online at http://www.perjournal.com
ISSN: 2148-6123
http://dx.doi.org/10.17275/per.25.32.12.3

Connecting Geometric Reasoning Skill and Self-Efficacy Perception Variables in terms of Cognitive and Perceptual Dimensions

Büşra Alphayta
Mathematics and Science Education, Trabzon University, Trabzon, Türkiye
ORCID: 0000-0003-1963-7936

Hayal Yavuz Mumcu*
Department of Mathematics and Science Education, Ordu University, Ordu, Türkiye
ORCID: 0000-0002-6720-509X

Article history
Received:
24.08.2024

Received in revised form:
14.11.2024

Accepted:
02.03.2025

Key words:
Geometric reasoning; scale development; self-efficacy perception; relational research; 8th-grade students

This study aims to develop a scale to determine eighth-grade students' self-efficacy perceptions towards geometric reasoning and to evaluate and relate the variables of geometric reasoning skill and self-efficacy perception in terms of cognitive and perceptual dimensions. The research was conducted in two stages. In the first stage, the validity and reliability studies of the developed scale were conducted, and its usability was demonstrated. In the second stage, geometric reasoning skill and self-efficacy perception variables were examined in terms of cognitive and perceptual dimensions. In the first phase of the study, the general survey method was used and in the second phase, relational survey method was used. The scale development phase of the study was conducted with 595 eighth grade students. In the second stage of the study, 40 students studying at the eighth-grade level were studied. As a result of this study in which Geometric Reasoning Self-Efficacy Scale, Geometric Reasoning Skill Test and Unstructured Interviews were used as data collection tools, it was seen that students' geometric reasoning self-efficacy perceptions were generally at medium level and their geometric reasoning skills were at low level. Considering the different dimensions, it was found that the mean scores of students' self-efficacy and skill scores related to the perceptual dimension were higher than the cognitive dimension. In addition, there is a positive, moderate and significant relationship between students' geometric reasoning skills and their geometric reasoning self-efficacy perceptions. Depending on the results obtained from the research, it is suggested that activities or learning environments be designed to improve students' geometric reasoning skills and that the effects of different learning environments on geometric thinking processes be investigated.

Introduction

Today, mathematical learning is regarded as a process due to its importance, and the components of this process are expressed differently in national and international curricula. The National Council of Teachers of Mathematics (NCTM), which is accepted all over the world in

* Correspondency: hayalym52@gmail.com

mathematics education and is considered an important reference in the creation of curricula of developed/developing countries, highlights the theme of learning mathematics as well as using mathematics in real life; in this context, it reveals mathematical skills as standards of today's learning processes. These skills are expressed in the NCTM (2000) document as problem-solving, reasoning (and proof), connecting, communication, and representation.

Mathematical skills are highlighted in parallel with NCTM in the mathematics curriculum in our country, while reasoning, connection, and communication (mathematical process skills) are emphasized as the main core of these skills (Ministry of National Education [MoNE], 2013). While many studies (Kaur & Lam, 2012; NCTM, 2000; Van de Walle, 2016) describe the process skills targeted to be acquired by students within the boundaries of school mathematics as integral parts of mathematical study processes, it can be said that mathematical reasoning, one of these skills, has gained more importance today than other skills. In fact, Ball and Bass (2003), Douek (2005) and Krummheuer (1995) state that mathematical reasoning is a more comprehensive skill that includes communication and connection. In different studies (Ball & Bass 2003; Sabirin, Aminah, Muhniansyah, & Atsnan, 2021; Sumarsih, Budiyono, & Indriati, 2018), the importance of mathematical reasoning skill in terms of the development of other skills is emphasized and, in this context, it is accepted that the relevant skill is at the focus of mathematical study processes. Similarly, it is seen that the skill at the center of the mathematical literacy process, which can be considered as the general goal of school mathematics today and which emphasizes the processes of using the mathematical knowledge students learn at school in their daily lives, has been updated as mathematical reasoning (Organisation for Economic Co-operation and Development [OECD], 2023). In the related research, although reasoning skill is addressed in the axis of problem solving, it is accepted that it includes more than problem solving processes in the context of evaluating interpretations and inferences about mathematical situations and forming arguments.

Mathematical reasoning, defined by Kaur and Lam (2012) as the ability to analyze mathematical situations and form logical arguments, is defined in the MoNE (2013) curriculum as "the process of obtaining new knowledge by using the tools (symbols, definitions, relationships, etc.) and thinking techniques (induction, deduction, comparison, generalization, etc.) specific to mathematics based on the information at hand" (p. v). Within the scope of the Programme for International Student Assessment (PISA) research, the related skill is defined as "the capacity to conceptualize real-life problems and situations and to use mathematical concepts, tools and logic to generate solutions to them" (MoNE, 2022, p.37). In the related study, it was stated that reasoning skill involves recognizing the mathematical structure inherent in a problem and developing strategies to solve this problem.

Kilpatrick, Swafford, and Findell (2001) defined mathematical competence as; i) 'conceptual understanding', which requires an understanding of mathematical concepts, operations and relationships; ii) 'procedural fluency', which includes the ability to carry out procedures flexibly, accurately, efficiently and appropriately; iii) 'strategic competence', which is the ability to formulate, represent and solve mathematical problems; iv) 'reasoning', which is the capacity to think logically, explain, justify and evaluate one's thinking; and v) 'productive disposition', which is an orientation towards seeing mathematics as logical, useful, valuable and reasonable. In the same study, it is stated that although all elements are important and interrelated for the concept of mathematical competence, reasoning processes are an important element that holds everything together and acts as a glue. This is because reasoning allows concepts and procedures to be connected in logical ways, suggests ways of solving problems, and allows disagreements to be resolved in logical ways. According to Ball and Bass (2003), reasoning is the "fundamental skill" (p. 28) of mathematics and is necessary for many different purposes, such as understanding mathematical concepts, using mathematical ideas and procedures flexibly, and constructing mathematical knowledge. Sarpkaya Aktaş (2020) states that mathematical reasoning is a skill that helps a student use all other mathematical skills and that the emphasis on reasoning should prevail in mathematical activities in order to see Mathematics as a powerful way of making sense of the world. According to NCTM (2000), reasoning skills are an integral part of doing mathematics and should be part of the curriculum at all levels, from pre-school to the end of secondary education. In this context, it can be said that reasoning skill is a very important skill that should be acquired by students within the scope of school mathematics and for this reason, it constitutes the subject of this study.

One of the fields in which reasoning is used intensively in mathematics education and is important in this context is geometry (Morsanyi, Prado, & Richland, 2018). Geometry, which forms the foundations of mathematical thinking, is defined as a complex network system in which geometric concepts, reasoning, and different representations used to conceptualize and analyze spatial situations or environments are interconnected (Battista, 2007; Tutan, 2019). Geometry, which is a unifying theme in the mathematics curriculum, is defined as an ideal tool that develops different ways of thinking such as problem solving, spatial and visual thinking, and reasoning, and enables the understanding, comprehension, and expansion of mathematical concepts (Goldenberg, Cuoco, & Mark, 1998, as cited in Köse, Tanışlı, Özdemir- Erdoğan, & Yüzügüllü-Ada, 2012). In this context, geometric thinking skill, which in the simplest sense means the use of reasoning skills in situations involving geometry, is defined by İlhan (2019) as follows: "In the event that an individual encounters a geometric shape, he/she can interpret this shape based on the information in his/her mind and presented to him/her, analyze the shape in a practical way by determining a new cognitive path or method, associate this shape with different problem situations and use it in problem solving processes" (p.24). Geometric reasoning can be considered high-level geometric thinking. As reasoning is an ability and skill that can only be used at advanced stages of thinking processes (Umay, 2003), the concept of reasoning is accepted as a higher-level form of mathematical thinking processes in different studies (Alkan & Taşdan, 2011; Yavuz Mumcu, 2019). Individuals with good geometric reasoning skills can reach the required information and make effective decisions by becoming aware of geometry in many areas of life. Additionally, these individuals can appreciate geometry in art, nature, and architecture. The fact that geometry is also used in science, technical, and professional fields due to the interdisciplinary feature of mathematics today has increased the importance of the geometric reasoning skill and made the development of this skill important (Kızıltoprak, 2020). There are many studies and theories conducted in this context. In these, the geometric reasoning process is generally expressed through developmental and cognitive approaches. Developmental approaches accept that there are hierarchical relationships between geometric reasoning processes and explain the development of the relevant levels for the individual based on the increase in the level of knowledge. Moreover, in these approaches, the transition between levels is an indicator of competence in geometry (Clements, 2003). Piaget and Van Hiele, who adopt the developmental approach, try to explain the nature of geometric reasoning with various steps. In addition, the cognitive approach accepts that geometric reasoning is carried out through cognitive processes. In this approach, there is no hierarchical relationship between these processes. Among these approaches, Fischbein's figural concept theory and Duval's cognitive model can be cited. The theoretical framework of this research is Duval's Cognitive model, one of the cognitive approaches, as it creates a more holistic framework, and this model is explained in detail below.

Duval's cognitive model

French researcher Raymond Duval discussed geometric reasoning in two different dimensions, i.e., perceptual and cognitive processes, and stated that competence in geometry depends on the interaction of these dimensions (Duval, 1995, 1998; Jones, 1998). The perceptual and cognitive processes put forward by Duval and defined by Güven and Karpuz (2016) are given in Figure 1.

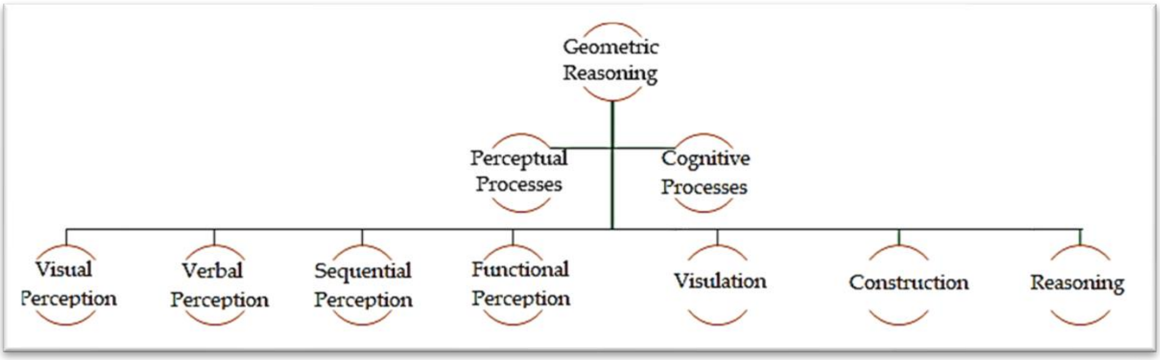


Figure 1. Duval's cognitive model

Perceptual processes

Perceptual processes, which consist of four perceptions (visual, verbal, sequential, functional perception) that have no hierarchical relationship with each other and are also called processes of looking at shapes, perform different functions that enable mathematical relationships on the shape to be recognized. The perception regarding recognizing and determining a shape at first glance is defined as visual perception, the perception regarding the process of establishing a relationship between mathematical principles and the shape at the inference stage based on the given information as verbal perception, the perception regarding the process of establishing or describing the formation of geometric shapes with the help of a tool as sequential perception, and the perception that provides insight for the solution of the problem by making changes to the initial shape (breaking it into pieces, changing its direction, drawing an auxiliary line or line segment, etc.) is defined as functional perception (Duval, 1998).

Cognitive processes

Stating that there are three independent cognitive processes for geometric reasoning (visualization, creation and reasoning), Duval expresses that each process performs different functions (cited by Güven & Karpuz, 2016). While the process of visualizing geometric objects is defined as visualization, the formation of geometric shapes with compasses, rulers, and dynamic geometry software and the sequencing of the formation process is defined as formation, while the replacement of the existing information with new information depending on the representation of the information is defined as reasoning (Duval, 1998). Accordingly, change or expansion in existing knowledge occurs through visual representations and each form of representation affects reasoning in different ways (Duval, 1998). Although these processes are independent of each other, they interact with each other, and the strength of this interaction is necessary to gain competence in geometry. The interaction between cognitive processes is given in Figure 2.

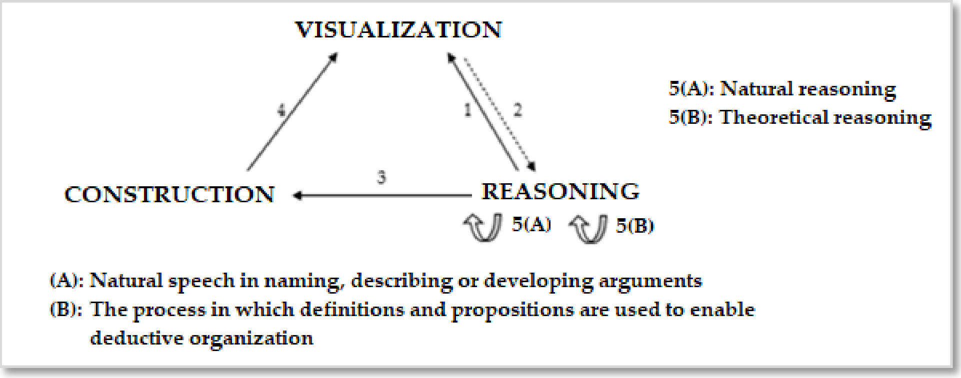


Figure 2. Interaction between cognitive processes (Duval, 1998)

The arrows in Figure 2 show the ways in which the processes support each other, and the reason for the dashed arrow from visualization to reasoning is that the visualization process does not always support the reasoning process. In addition, since a geometric shape is visualized at the time of its formation, there is no transition from the formation process to the direct reasoning process. The arrows specified in 5(A) and 5(B) indicate that the reasoning process can occur independently of the formation and visualization process. In this context, Duval (1998) states that the main problem of school mathematics is that students cannot acquire skills related to cognitive processes in relation to each other. Therefore, these independent cognitive processes need to be developed separately in order to solve this problem. At this point, it is thought that studies that include the sub-dimensions put forward by Duval regarding geometric reasoning skills and that will be conducted to develop these dimensions will contribute to the field. For this reason, Duval's theoretical framework was used in this study.

Purpose of the study

Studies in the literature show that students generally exhibit low performance in geometric reasoning processes (Alaylı, 2012; Berkant & Çadırlı, 2019; Fidan & Türnüklü, 2010; Karakarçayıldız, 2016; Michael, Gagathis, Avgerinos, & Kuzniak, 2011; Senk, 1989; Usiskin, 1982). Therefore, there is a need to improve student performance in this regard. When it comes to improving individual performances, affective factors, which are an important component of learning environments, have a special importance especially in mathematics classrooms. Self-efficacy (Zimmerman, 1995), which is a concept related to judgments about one's ability to perform a behavior and accomplish a task, is highly effective on individuals' performance. Studies show that there are positive and significant relationships between students' mathematics achievement and their self-efficacy perceptions (Gündoğdu, 2013; Pietsch, Walker, & Chapman, 2003). From this point of view, it is important to investigate students' self-efficacy perceptions about geometric reasoning. However, there is no comprehensive and effective measurement tool for geometric reasoning self-efficacy perception in the literature and it is seen that there is a need for a measurement tool that measures students' self-efficacy beliefs towards geometric reasoning. Accordingly, in the first stage of this study, it was aimed to develop a valid and reliable measurement tool for the geometric reasoning self-efficacy beliefs of 8th grade students. The research question that will be tried to be answered at this stage is; "Which dimensions do 8th grade students' geometric reasoning self-efficacy perceptions consist of?". It is thought that the development of a tool that measures the perception of self-efficacy for geometric reasoning will contribute to new research on the subject. The developed scale can be used in experimental studies targeting the development of geometric reasoning processes, and the factors affecting self-efficacy perception towards geometric reasoning can be revealed with the help of this scale. In this context, it is thought that the geometric reasoning self-efficacy scale will contribute to the examination and development of students' geometric reasoning skills and processes.

Self-efficacy belief, which is one of the important factors in the affective domain of mathematics teaching, is accepted as a predictor of individuals' performance in mathematics (Pajares & Graham, 1999). Therefore, it is foreseen that students' performance and self-efficacy beliefs towards geometric reasoning are interrelated processes. In addition, students' performances in the sub-dimensions of geometric reasoning processes are important for the development of this skill. In this context, in the second stage of the study, it is aimed to investigate the relationships between geometric reasoning skills and self-efficacy perceptions of 8th grade students. At this stage, the following questions were tried to be answered.

- What is the level of 8th-grade students' geometric reasoning skills in cognitive and perceptual dimensions?
- What is the level of 8th-grade students' geometric reasoning self-efficacy perceptions in cognitive and perceptual dimensions?
- Is there a statistically significant relationship between 8th-grade students' geometric reasoning skills and self-efficacy perceptions?

Method

The methods used in the first and second stages of this research, are presented separately below.

First stage

In the first stage of the research, the general survey model was used in terms of its suitability for the purpose of the study. The general survey model is "the survey arrangements made on the whole universe or a group, sample or sample to be taken from it, in order to make a general judgment about the universe consisting of a large number of elements" (Karasar, 2004, p.4). Since it was aimed to determine students' geometric reasoning self-efficacy perceptions and the sub-dimensions of the variable in question, it was deemed appropriate to use this method.

Research group

In the first stage, 605 students studying in the eighth-grades of seven different public schools were studied. Simple random sampling method was used to determine these schools. The students in each school were informed about the research process and the research was conducted with 605 students volunteered to participate in the research. The data obtained from the 595 students were analysed after the ones that could not be used within the scope of the research (those who filled in one side of the form but not the other, those who marked all the questions the same) were removed from the data obtained. While determining the number of students in the sample, reference was made to the studies stating that a sample of at least five times the number of items in the scale is sufficient (Tavşancıl, 2005). The number of draft scale items developed within the scope of the research is 44. In this context, it was accepted that the sample in the scale development process was of sufficient size. In the scale development process, 320 of the data obtained from 595 students were used within the scope of Exploratory Factor Analysis (EFA) and 275 of them were used within the scope of Confirmatory Factor Analysis (CFA).

Research application permits

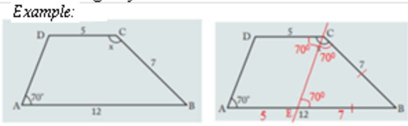
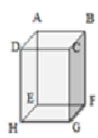
In conducting this research, the necessary permissions were obtained from Ordu University Social and Human Sciences Research Ethics Committee with the decision dated 11.03.2020 and numbered 2020-17, and from Samsun Provincial Directorate of National Education with the decision dated 04.03.2021 and numbered E-92596593.

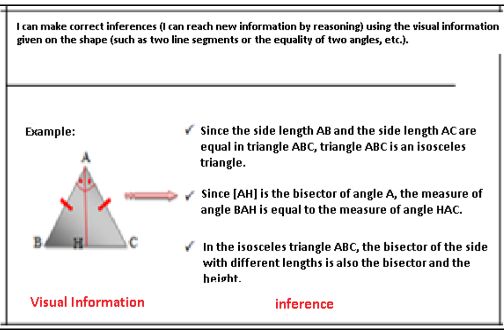
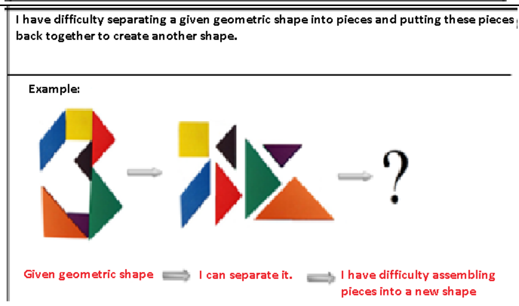
Process: Development of the scale

In the development process of the Geometric Reasoning Self-Efficacy Scale (GRSS), the eight stages suggested by De Vellis (2003) were followed. Accordingly, in the study process, (i) what is to be measured was clearly determined, (ii) the theoretical structure of the variable to be measured and the related variables were presented in detail, (iii) the item pool was created, (iv) the format of the measurement tool was decided, (v) the scale items were reviewed by experts, (vi) item validity was ensured, (vii) the scale was applied, and (viii) the items were evaluated, and the scale was finalised.

Creation of the item pool. During the scale development phase, an item pool was first created. While creating the item pool, Duval's (1995) theoretical framework and the indicators put forward by Karpuz (2018) for geometric reasoning processes were taken into account. The items created and related indicators are exemplified in Table 1.

Table 1. GRSS-scale items-relevant dimensions and indicators

Indicator	**Item No.**	**Reasoning Dimension/Sub-Dimension**	**Item**
The changes made to the shape can be explained, but the changes cannot be justified using the necessary definitions, axioms, and theorems.	31	**<u>Cognitive Dimension</u>** Reasoning *Natural Reasoning*	I can explain the changes made to the figure, but I have difficulty understanding why. *Example:*  I may notice that to find the measure of the angle x in the trapezoid ABCD, the line CE is drawn **<u>parallel to [AD]</u>**. But I have difficulty understanding why this is done.
Definitions and theorems are used when making inferences, and no conclusion can be reached based on the appearance of the shape.	20	**<u>Cognitive Dimension</u>** Reasoning *Theoretical Reasoning*	When solving a geometry problem, I can reach a solution using definitions and theorems instead of **starting from the appearance of the shape**. <u>Example</u>:  What integer values can x take in triangle ABC? In solving this problem, starting from the appearance of the shape, instead of **ABC is a right triangle since m (B) = 90°. Therefore x = 5.** rather, since the measure of angle B is not given as 90°, I can prefer and apply the following solution. **"Using the triangle inequality, 1<x<7. Accordingly, the integer values that x can take are 2, 3, 4, 5 and 6."**
The given geometric shape and the basic geometric elements that make up the shape are realized and can be named.	9	**<u>Perceptual Dimension</u>** *Visual perception*	I have difficulty recognizing the basic geometric elements (edges, corners, angles...) that make up the shape. <u>Example</u>:  The rectangular prism : Details: ? (I have difficulty noticing) Corners: ? (I have difficulty determining) Height: ? (I have difficulty determining) Side faces: ? (I have difficulty determining)

<table>
<tr><td>Correct inferences can be made by translating visual information on a shape (equality of two-line segments or two angles, etc.) into verbal information.</td><td>10</td><td><u>Perceptual Dimension</u>
• Verbal Perception</td><td>
</td></tr>
<tr><td>A given geometric shape can be divided into pieces and these pieces can be reassembled to create another shape.</td><td>15</td><td><u>Perceptual Dimension</u>
• Functional Perception</td><td>
</td></tr>
</table>

Five-point Likert answer options (‘Always’, ‘Mostly’, ‘Sometimes’, ‘Rarely’, and ‘Never’) were used to answer the items in the scale. Expert opinions were consulted to determine whether the draft scale containing self-efficacy items was suitable for the aim of the research. For this purpose, the scale items were finalized by taking the opinions of three faculty members who are experts in the field of mathematics education, one faculty member who is an expert in the field of measurement and evaluation, and three mathematics teachers working in different secondary schools. In this process, some scale items were removed from the scale, or the expressions were changed.

Pilot application. Before the application, a pilot application was carried out to determine whether the draft scale items were understandable by the students and whether there were any items that did not serve the purpose. Within the scope of the pilot application, 15 students studying at the researcher's school and with different levels of academic success were included. The draft scale items developed within the scope of the research were applied to the selected students within one class session, and during this process, the students were informed about how to answer the scale. At the end of the pilot application, it was determined whether there were any defective items in the scale, and the scale was given its final form in the light of this data, and a total of 44 draft scale items, of which 24 were positive and 20 were negative, were determined.

Validity and reliability analyses. Scope, face, criterion, and construct validity studies were conducted for the validity of the scale developed within the scope of the research. Examples can be shown such as literature review for scope and face validity, theoretical structure constructed, and expert opinions. In order to determine criterion validity, the correlation coefficient between the final scale and the results of the Geometry Self-Efficacy Scale developed by Cantürk-Günhan and Başer (2007) was examined. For construct validity, item analyses planned in two stages were first conducted, and then exploratory and confirmatory factor analyses were conducted.

Item analyses were conducted based on the difference between lower and upper group averages and correlation. After the item analyses, the factor structure of the scale was examined using exploratory factor analysis (EFA). In the EFA, principal components analysis was used as the factorization method, and the criteria for the number of factors were the eigenvalue greater than 1 (Kaiser rule) and the explained variance over 5%. For an item to be included under a factor, the factor loading must be at least .30 (Büyüköztürk, 2005). Then, the Oblimin Oblique Rotation (Seçer, 2015, p.169) technique was used to combine interrelated items under different factors, since it is assumed that the sub-factors in the scale are related to each other.

Confirmatory factor analysis (CFA) was applied to test whether the data was compatible with the factor structure acquired by EFA. The results as a result of CFA were evaluated considering fit indices. The reliability of the scale developed in this research was ensured by calculating the Cronbach alpha internal consistency coefficient. This coefficient is "fairly reliable" between .60 and .80, and "highly reliable" between .80 and 1 (Akgül and Çevik, 2003). GRSS developed at the end of the process consists of a total of 20 items: 14 positive and six negative. 20 items of the scale were seen to be distributed into two factors. While naming the factors, the common emphasis of the items and the theoretical framework were taken into consideration. The first factor consisting of 14 items, is called Perceptual Reasoning and the second factor consisting of six items is called Cognitive Reasoning.

Data analysis

The Geometric Reasoning Self-efficacy Perception Scale (GRSS) was developed to determine the geometric reasoning self-efficacy perception levels of the students participating in the study. Positive items in the scale are scored as 5-4-3-2-1, and negative items are scored as 1-2-3-4-5. Accordingly, the lowest score on the scale is 44 and the highest score is 220, high scores are accepted as indicators of strong geometric reasoning self-efficacy perception, and low scores are accepted as indicators of weak geometric reasoning self-efficacy perception.

Within the scope of the first sub-problem of the research, the scale arithmetic average score was calculated for each student, by dividing the total score from the scale by the number of items in the scale (Kan, 2009, p.407). In order to determine the level of this value in the self-efficacy perception scale, the group range coefficient was determined as (5-1) /3=1.3 and the self-efficacy perception level score ranges were as follows: 5.00-3.68: High level, 3.67-2.35: Medium level, and 2.34-1.00: Low level.

Second stage

In the second stage of the research, the relational survey model was used. Relational survey is a research model that aims to determine the existence or degree of change between at least two variables (Karasar, 2004). In this research model, the existence of a relationship between variables is examined without intervening in the variables. In the second stage of this research, since students' self-efficacy perceptions and skills towards geometric reasoning will be related in different dimensions, it was deemed appropriate to use this model.

Study group

In the second stage of this research, a total of 40 students attending 8th grades of the researcher's own school were studied. In the selection of these students, the convenience sampling method was preferred, which allows the researcher to select a situation that is close to him/her and easy to access (Kılıç, 2013). Attention was also paid to the fact that the determined students volunteered to take part in the study.

Data collection tools

In the second stage of this study, two different data collection tools were used. These are the Geometric Reasoning Self-Efficacy Perception Scale (GRSS) developed by the researchers within the scope of the study and the Cognitive Process Tests developed by Karpuz (2018). Within the scope of this study, the relevant data collection tool was named/called as/entitled Geometric Reasoning Skill Test (GRST).

Geometric reasoning self-efficacy scale (GRSS). In the second stage of the study, the reliability analyses of the GRSS developed in the first stage were repeated. Accordingly, the Cronbach's alpha internal consistency coefficient for the whole scale was calculated as 0.934, the alpha coefficient for the perceptual reasoning factor was 0.920, and the alpha coefficient for the cognitive reasoning factor was 0.876.

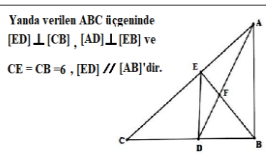
Geometric reasoning skill test (GRST). It consists of two different cognitive process tests: (I) Looking at Shapes and (ii) Reasoning. Among these tests developed by Karpuz (2018), there are six questions in the Looking at Shapes cognitive process test and five questions in the Reasoning cognitive process test. One question in the Reasoning cognitive process test was removed from the test as it was outside the 8th-grade achievements. Therefore, these tests administered to the students in the study group consist of a total of 10 questions. Within the scope of this research, the cognitive process test of looking at the shape was named as Perceptual Reasoning Test and the cognitive process test of reasoning was named as Cognitive Reasoning Test.

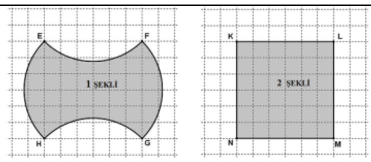
Data analysis

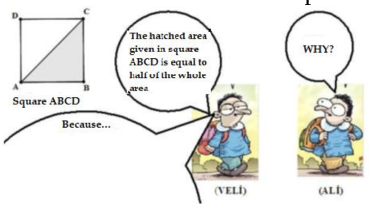
In the data analysis process carried out in the second stage of the study, the Categorical Scoring Rubric developed by Karpuz (2018) and Unstructured Interviews were used.

Categorical scoring scale. The categorical scoring scale was used to convert the students' responses to the questions in the GRST into quantitative data and thus to determine their geometric reasoning skill levels. In the categorical scoring scale, there are different scoring categories for each question in the GRST. Accordingly, the sample questions in the test and the related scoring criteria are given in Table 2.

Table 2. Some questions and scoring criteria in GMBT

TD	***Question***	***Indicator***	***Scoring***
Visual Perception	 Q1. How many dimensions has the geometric figure given above?	✓ Can tell the size of the given geometric shape.	0: Writes the size of the given shape incorrectly or leaves the question blank. 1: Writes the correct size of the shape.
	Q2. Which geometric shapes does the figure given in the question consist of?	✓ Recognises the the basic geometric elements that make up the shape and	0: Does not write any shape or leaves it blank.

		can name them.	1: Writes only triangles or only quadrilaterals. 2: Writes different geometric shapes.
Functional Perception	 Which of the following is correct about the areas of the 1 and 2 hatched shapes given above on unit squares? Explain with the reason. a) The area of figure 1 is greater than the area of figure 2. b) The area of figure 1 is smaller than the area of figure 2. c) The areas of figures 1 and 2 are equal. d) No comment can be made since no length is given.	✓ Can decompose a given geometric shape into parts and use these parts to create another shape. ✓ Can focus on some parts of the shape and change the shape by adding or deleting new geometric objects. ✓ Can change the position and orientation of the given shape or its sub-parts. ✓ Does not need numerical data to make changes on the given geometric shape.	0: Leaves it blank, marks the wrong option or makes a wrong explanation. 1: Reaches the correct answer by measuring or marks the correct option but does not explain. 2: Reaches the correct answer by adding or subtracting on the figure.

Theoretical Reasoning	Below are the conversations between two students about the square ABCD. Try to complete Veli's speech.  	✓ Uses definitions and theorems when making inferences. Does not reach a conclusion based on the appearance of the figure.	0: Leaves it blank or writes an incorrect explanation - Responds by being influenced by the appearance of the figure. 1: Makes correct explanations by using colloquial language while making inferences. 2: Makes correct inferences by using mathematical principles.

TD: Test dimension

The lowest score that can be obtained from the Perceptional reasoning test is 0 and the highest score is 14, while the lowest score that can be obtained from the Cognitive reasoning test is 0 and the highest score is 9. The lowest score that can be obtained from the GRST, which consists of a total of 10 questions, is 0 and the highest score is 23. For making a decision on the geometric reasoning skill levels of the students at this stage, the group range coefficient was calculated as 23/3 = 7.6 and the score ranges were as follows: 0- 7.6: Low level, 7.7-15.3: Medium level, and 15.4- 23: High level.

Unstructured interviews. The unstructured interviews conducted within the scope of the research were used to reveal the reasons for the answers given by the students in the second stage to the questions in the GRST and to place these answers in the correct category according to the categorical scoring scale. For this purpose, interview processes were conducted with 11 students and audio recordings were taken during the interviews. The data analysis process carried out in this part of the research is exemplified below.

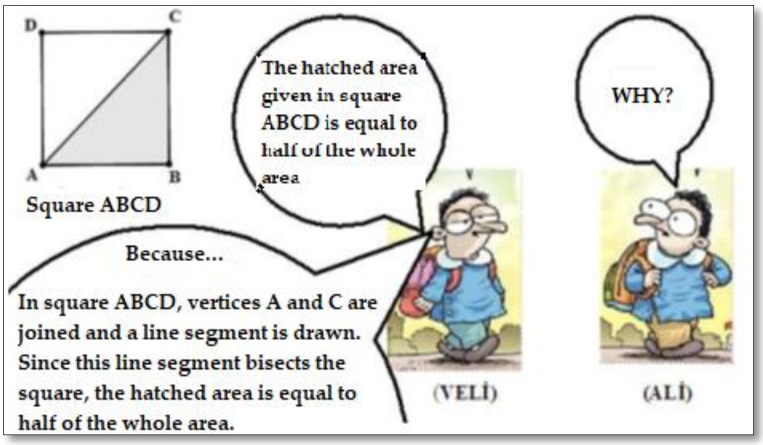


Figure 3. Written answer of student coded S7 to the second question of the GRST

On examination of the written answer, the student was seen to think that the line segment (diagonal) connecting corners A and C divided the square into two equal parts. It is thought that the student makes a decision based on the appearance of the shape and that his intuition is effective in giving such an answer. However, during the interview with the student, it was seen that the student made correct judgments about the shaded area without referencing the appearance of the shape. The relevant part of the interview process is given below.

> Researcher: You say that corners A and C are joined, and a line segment is drawn, and this line segment divides the square into two equal parts. So how did you decide on this?
>
> Student: Sir, since they are corners, I thought they would be two equal pieces when placed together.
>
> Researcher: Why do you think so, how do you decide that?
>
> Student: Sir, they would be the same if folded.
>
> Researcher: So, if it is folded on top of each other, does it become a match?
>
> Student: I think so, sir.

As can be seen from the interview section, the student used colloquial language instead of mathematical language in his statement. Accordingly, the student uses words, expressions, and subjective meanings that he/she interprets in his/her own way, instead of mathematical concepts. When the answer is examined, the student is understood to think that the diagonal of the square divides the area of the square into two equal parts due to the overlapping of the triangles formed when the square is doubled along its diagonal. The relevant answer of the student, who references the folding process along the diagonal instead of the appearance of the shape, corresponds to the indicator 'Correct explanations are made using colloquial language when making inferences on the shape' in the categorical score card determined for the question. Therefore, the relevant answer of the student was coded as '1' by associating it with the indicator in question.

Prior to the data analysis process carried out to answer the third and fourth sub-problems of the research, normality tests were performed for the students' self-efficacy (scale) and reasoning scores (see Table 3).

Table 3. Reasoning and self-efficacy scores normality tests

	Skewness		Kurtosis		Standard scores	Error	Z Kolmogorov-Smirnov		
	Statistic	Std. Error	Statistic	Std. Error	$Z_{skewness}$	$Z_{kurtosis}$	Statistic	df	p
Geometric Reasoning Self-Efficacy Score	-.597	.374	-.300	.733	-1.60	-.40	.108	40	.200
Geometric Reasoning Skills Score	1,195	.374	1,107	.733	3.20	1.5	.157	40	.014

As a result of the tests, the geometric reasoning self-efficacy perception scores were observed to be normally distributed, but the geometric reasoning skill scores were observed not to meet the normality assumptions ($Z_{skewness}$= 3.2, $p<0.05$). Accordingly, 'Spearman Rank Difference Correlation Calculation' was used to examine the relationship between students' self-efficacy perception scores and reasoning skill scores.

Results

Results from the first stage

Item analysis results

As a result of the item analysis based on the difference in lower-upper group averages, it is p>.05 for the 40th item, and due to the absence of a significant difference between the lower and upper group averages, this item was removed from the scale. As a result of the item analysis based on item-total correlation, items 18, 26, 31, 36, and 41 with correlation values lower than .30 were removed from the scale. Item analysis results are given in Table 7.

Exploratory factor analysis results

Before starting the exploratory factor analysis, the Kaiser-Meyer-Olkin (KMO) coefficient was examined to test for the presence of a sufficient sample size to perform the factor analysis, and Bartlett's test of Sphericity results were examined to determine whether the data came from a multivariate normal distribution. According to these values, it was seen that the sample size was sufficient to perform factor analysis (KMO = .947) (Hutcheson & Sofroniou, 1999); Bartlett test Chi-square value was found to be statistically significant (X^2=7460.758; p<.001). In the factor analysis carried out within the scope of the research, direct oblimin rotation was utilized and eigenvalue, the percentage of contribution to the total variance, and scree plot (Tabachnick & Fidell, 2007; Tavşancıl, 2005) were used as criteria in determining the number of factors. According to the first results as a result of the principal components analysis performed on the items of the scale, the scale items were seen to be distributed into five factors with eigenvalues above 1.00 (see, Table 4).

Table 4. EFA first phase explained variance rates

Dimensions	Eigenvalue	Variance	Cumulative variance
Factor 1	14.645	38.540	38.540
Factor 2	4.151	10.923	49.463
Factor 3	1.644	4.327	53.790
Factor 4	1.271	3.345	57.135
Factor 5	1.025	2.697	59.833

According to the line graph drawn based on the eigenvalues of the factors (Fig.4), a rapid decrease was seen until the third factor, and the slope plateaued after this point. Therefore, it is accepted that the contributions of the factors to the variance after this point are quite small.

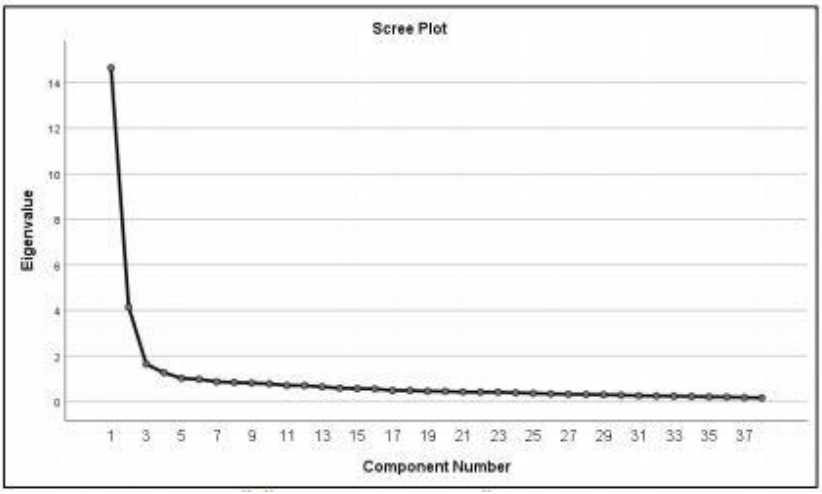


Figure 4. GRSS factor-eigenvalue line graph

Considering the two-dimensional feature of the theoretical structure of the research in the form of cognitive processes and perceptual processes, EFA was re-applied at this phase of the research by limiting it to two factors. The results of the EFA, which was reapplied for a two-factor structure, were evaluated in terms of loading values and overlap, and accordingly, a total of 18 items (3, 6, 8, 12, 13, 14, 15, 17, 20, 22, 24, 28, 30, 32, 33, 35, 37, 44) were removed from the scale and the same rotation process was repeated. At the end of the process, a two-factor structure was obtained with the total amount of variance explained by 52.384% (see, Table 5).

Table 5. Variance rates explained by the sub-dimensions of GRSS

Dimensions	Eigenvalue	Variance	Cumulative variance
Factor 1	8.207	41.037	41.037
Factor 2	2.269	11.347	52.384

After all these analyses, the results regarding the factor loadings of all items of the final scale are given in Table 6.

Table 6. GRSS- factor loading values

Items	Factor Loading Values	
	Factor1	**Factor2**
Item 1	.548	
Item 2	.655	
Item 4	.610	
Item 5	.718	
Item 7	.695	
Item 9	.733	
Item 10	.734	
Item 11	.706	
Item 16	.702	
Item 23	.695	
Item 25	.638	

Item 27	.662	
Item 29	.694	
Item 38	.498	
Item 19		-.841
Item 21		-.861
Item 34		-.777
Item 39		-.821
Item 42		-.809
Item 43		-.770

According to these results, it can be seen that the factor loading values of the scale items range from .861 to .498. This indicates a quite high relationship between the items with their respective factors. In this direction, Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz and Demirel (2018) state that the factor loading value expresses the correlation between the variable and the factor, and regardless of its sign, loading values of .60 and above are large sizes, and loading values between .30-.59 are medium sizes. As a result of EFA, items 1, 2, 4, 5, 7, 9, 10, 11, 16, 23, 25, 27, 29 and 38 were arranged under the first factor; Items 19, 21, 34, 39, 42 and 43 were collected under the second factor. Accordingly, depending on the common emphasis of the expressions under the factors and the theoretical structure created in the research, the first factor consisting of 14 items was named Perceptual Reasoning, and the second factor consisting of 6 items was named Cognitive Reasoning. After EFA, a structure consisting of 20 items and two factors in total, six of which were negative, was constituted.

Results of the confirmatory factor analysis

In order to verify the structure revealed as a result of EFA for the GRSS, the data set was loaded into the LISREL statistical program, and the covariance matrix was prepared accordingly. According to the CFA results, the Chi-Square (X^2) goodness was seen to be a perfect fit considering the number of samples (X^2=308.93; df=167; X^2/sd=1.849). According to the results of the CFA, modification indices were examined to understand whether the model needed improvement; and it was deemed appropriate to make modifications between articles 23-27 and 27-38. Fit indices calculated by confirmatory factor analysis were as follows: RMSEA = .056; CFI= .94; GFI= .90; AGFI= .87; IFI= .94; NFI=.89; NNFI=.94. Therefore, it can be said that the data are well-compatible with the structure developed. The diagram representation of the standardized analysis values for the measurement model of the data analysis is as follows (Figure 5).

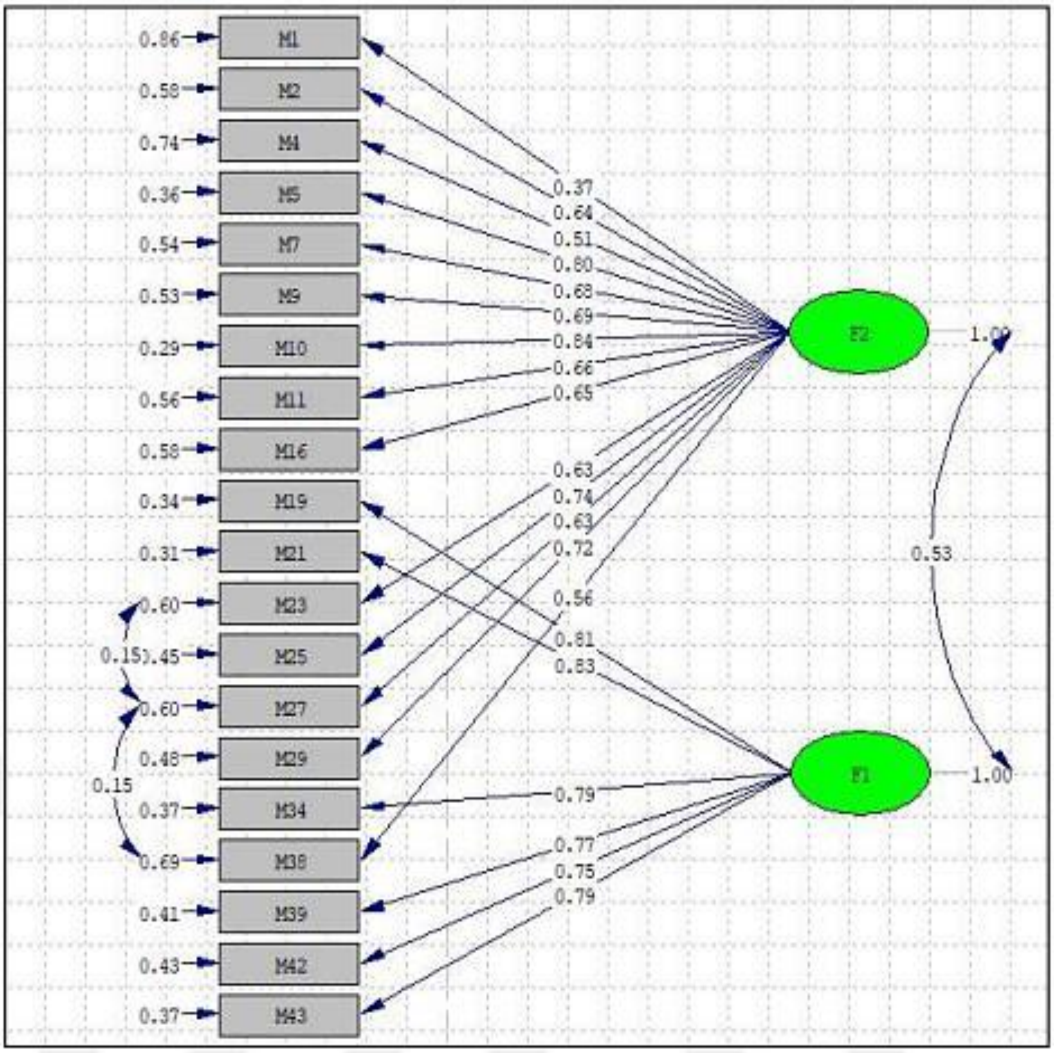


Figure 5. GRSS-confirmatory factor analysis model

The standardized coefficients in the path diagram obtained as a result of the CFA reveal evidence of the extent to which the scale items can represent their related factor (Şimşek, 2007, p.85). Additionally, all t-values were observed to be greater than 1.96 ($p<.05$) and were significant (Jöreskog & Sörbom, 1996). Accordingly, these items can be said to make a significant contribution to the latent variables of GRSS. These data indicate acceptable internal consistency of the sub-dimensions created for GRSS in the research sample.

Results of the criterion validity

A different scale was used to measure geometric reasoning self-efficacy (or an equivalent phenomenon) was applied to the participating students in order to determine the criterion validity of the GRSS. In this context, the Geometry Self-Efficacy Scale developed by Başer and Cantürk-Günhan (2007) was applied to 40 students who participated in the second phase of the research. The Cronbach-alpha reliability value of the relevant scale is 0.90. Within the scope of this research, the relevant value was calculated as 0.87. At the end of the application process, the Pearson correlation coefficient between the Geometry self-efficacy scale and the GRSS was calculated as 0.773 ($r=0.773$, $p<0.01$). This result indicates that the correlation value between the scale scores is high and statistically significant (Büyüköztürk et al., 2018).

Results of the scale reliability

The results of the analysis carried out for the reliability of GRSS are given in Table 7.

Table 7. GRSS-item analysis and results of reliability

Factor	Item No	Item-Total Score Correlation	t (Lower 27%-Upper 27%)	Cronbach's Alpha
Perceptual Reasoning	1	.398	-6.630**	
	2	.583	-13.476**	
	4	.519	-12.512**	
	5	.751	-21.794*	
	7	.592	-13.997**	
	9	.668	-15.601**	.904
	10	.758	-22.276**	
	11	.644	-15.147**	
	16	.636	-14.782**	
	23	.642	-16.832**	
	25	.730	-23.472**	
	27	.632	-18.922**	
	29	.648	-17.716**	
	38	.517	-12.515**	
Cognitive Reasoning	19	.681	-21.018**	
	21	.686	-22.525**	
	34	.701	-22.675**	.897
	39	.654	-18.787**	
	42	.683	-22.712**	
	43	.678	-21.401**	
Overall Scale				.882

**P< 0.001

On examination of the values in Table 7, it is seen that the item-total score correlations are over 0.30, the t-values obtained for the upper and lower groups are significant at the .001 level, and the internal consistency reliability coefficients for the sub-factors are above 0.80. Based on all these, the items and sub-factors in the GRSS can be said to create a highly reliable structure.

The correlation, arithmetic average, and standard deviation values between the GRSS factor scores are given in Table 8.

Table 8. Correlation, arithmetic average, and standard deviation values between GRSS factor scores

	F1	F2	Total	$\bar{X}$	Sd
Perceptual Reasoning (F1)	1			57.94	9.72
Cognitive Reasoning (F2)	0.601**	1		21.82	6.08
Total	0.940**	0.838**	1	79.76	14.22

N=595, **p<0.01

On examination of Table 8, the correlation values have a high level of relationships between the factors of the developed scale and the scale total, and a medium level of relationships between the factors of the scale. All these data show the confirmation of the hypotheses regarding the accepted structure for GRSS.

Results from the second stage

The findings obtained regarding the geometric reasoning self-efficacy perceptions of the students in the second stage of the study are given in Table 9.

Table 9. Students' GRSS scores

Sub-Factors	N	$\bar{X}$	SD	Cronbach's Alpha
Perceptual Reasoning	40	3.74	.85	.920
Cognitive Reasoning	40	3.40	.99	.876
Overall Scale	40	3.63	.82	.934

When the data in Table 9 were analysed, it was seen that the students' geometric reasoning self-efficacy perception levels were high in the perceptual reasoning sub-dimension, medium in the cognitive reasoning sub-dimension and in the overall scale. The findings obtained regarding the geometric reasoning skills of the students in the second stage of the study are given in Table 10.

Table 10. Descriptive statistics for cognitive process tests

	N	Min.	Max.	Process Test Average Scores
Perceptual Reasoning	40	0	14	3,425
Cognitive Reasoning	40	0	9	1,325
Overall scale	40	0	23	4.75

When Table 10 is analysed, it is seen that students' geometric reasoning skills are at low levels both for sub-processes and in general. The results of the correlation analysis conducted to determine whether there is a relationship between eighth-grade students' geometric reasoning skills and their geometric reasoning self-efficacy perceptions are presented in Table 11.

Table 11. Correlation analysis results for reasoning and self-efficacy perception variables

	Reasoning Score	Self-Efficacy Score
Reasoning Score	1.0	0,503
Self-Efficacy Score	0,503	1.0

As a result of the analysis, a positive, significant, and moderate relationship ($r = 0.503$, $p < 0.01$) was determined between geometric reasoning skills and self-efficacy perceptions. Accordingly, 25% of the change in students' geometric reasoning scores can be said to be explained by their geometric reasoning self-efficacy perception.

Discussion and conclusion

In this study, firstly, Geometric Reasoning Self-Efficacy Scale was developed by the researchers through validity and reliability studies. This scale was applied to the students in the research group and the level of students' geometric reasoning self-efficacy perceptions was determined. Accordingly, it was observed that the students' geometric reasoning self-efficacy perception levels were high in the Perceptual reasoning dimension and medium in the Cognitive reasoning dimension and the overall scale. Therefore, students' self-efficacy perceptions for geometric reasoning can be said to be higher in the perceptual dimension than in the cognitive dimension. This result shows that students' self-efficacy perceptions in perceiving a geometric situation are higher than their self-efficacy perceptions in reasoning and executing operations in the same situation. Therefore, students' self-efficacy perceptions and self-confidence in general in mathematical/geometric thinking and operation processes can be stated to have decreased. It is possible to find different studies supporting this situation in the literature (Berkant & Çadırlı, 2019; Bostancı, Kuzu, & Sıvacı, 2020). On examination of the literature, the research conducted on the subject seems to be rather related to the self-efficacy perception variable for geometry than the self-efficacy perception for geometric reasoning. Among the aforementioned line of literature, Berkant and Çadırlı (2019) and Ünlü (2014) stated that eighth-grade students' self-efficacy perceptions towards geometry were high or above average. In Erkek and Işıksal Bostan's (2015) research, it was stated that although the students' geometry achievements were low, their self-efficacy perceptions towards geometry were at a medium level. Different studies (e.g., Kaba, Boğazlıyan, & Daymaz, 2016; Yenilmez & Korkmaz, 2013) revealed that students' self-efficacy perception levels towards geometry were at a medium level, however, as the grade level increased, these levels decreased. Therefore, to the accumulated body of research generally finds middle school students' self-efficacy perception levels towards geometry to be at medium or high levels. In this context, the finding of the students' geometric reasoning self-efficacy perceptions at a moderate level in the present research can be interpreted as a consistent result with the literature. On the evaluation of this result considering the learning environments, it shows that although the students are partially confident about geometry courses, they cannot attain very high academic levels in geometry. In other words, students' awareness of their self-skills in geometry can be stated to be low. Despite the possibility of many different reasons for this situation, one of the reasons can be expressed as the fact that the gains in the field of geometry learning in the secondary school mathematics curriculum are generally supportive of perceptual processes and that students encounter attempts in this direction more frequently in courses. In this context, it is important for teachers to carry out teaching by taking into account the affective characteristics of students, especially in mathematics and geometry courses. For this purpose, it is recommended that teachers determine students' cognitive and affective readiness, plan teaching accordingly, and prevent students' unsuccessful learning experiences and situations that harm their self-confidence. Activities prepared accordingly can be presented gradually from easy to difficult, giving students a sense of achievement and thus developing the feeling of 'I can succeed' in students. Courses conducted accordingly will help students be motivated for the course and reach higher academic levels (Anıkaydın & Elitok Kesici, 2017; Biber, 2012).

Another result within the scope of the research is related to students' geometric reasoning skills.

As a result of the analysis, it was determined that the students showed low performance in both the perceptual and cognitive dimensions of the geometric reasoning skill test; however, their perceptual reasoning score averages were higher than cognitive reasoning. The results obtained here are compatible with the results obtained for the variable of self-efficacy. More clearly, students' self-efficacy and skill mean scores for the perceptual dimension are higher than those for the cognitive dimension. However, geometric reasoning self-efficacy perception is generally high, and geometric reasoning skills are low. An interpretation considering all data indicates that all the data obtained as a result of this research draws attention to the same point. In all cases, students' self-efficacy and skill levels remain higher in the perceptual dimension than in the cognitive dimension. According to Duval (1995), the strength of the interaction between perceptual processes and cognitive processes is important for high-level reasoning. On evaluation of the research results in this context, the interaction between the two processes can be stated to be weak and therefore the students in this research do not have high-level geometric reasoning skills.

When international studies on the subject are examined, in many of these, the concept of geometric reasoning is seen to be examined at different grade levels and mainly in the dimension of perceptual processes (Michael et al., 2011; Michael, 2013; Michael-Chrysanthou & Gagatis, 2013). In their research examining the sequential and functional perceptions of 9th- and 10th-grade students, Michael et al. (2011) concluded that 10th-grade students were more successful in the relevant processes. In his study examining the perceptual reasoning processes of high school students at different grade levels, Michael (2013) stated that the relevant processes developed as the grade level increased. However, in the research of Michael-Chrysanthou and Gagathis (2013) examining the functional perceptions of 9th and 10th-grade students, which is one of the perceptual reasoning processes, unlike other studies, no statistically significant difference was determined between the grade levels in said perception levels. This can be interpreted as perceptual processes may not always develop during the transition from one grade level to another. In addition to the studies examining geometric reasoning in terms of perceptual processes, there are also researches examining shape-concept interaction (Fischbein & Nachlieli, 1998). In their study aiming to determine students' geometric reasoning skill levels by looking at their ethnomathematics-based learning self-efficacy, Damaryanti, Mariani, and Mulyono (2017) concluded that students' ability to complete the geometric reasoning ability test was low, regardless of their self-efficacy levels. Therefore, it is seen that the results obtained are compatible with the results of this research.

On examination of the national literature on geometric reasoning skill, it is seen that different theoretical frameworks are used for the relevant concept, and the bulk of research on geometric reasoning with a cognitive approach are mainly conducted through shape-concept interaction (Güzeller, 2018; Karpuz, Koparan, & Güven, 2014; Mutluoğlu, 2019; Mutluoğlu & Erdoğan, 2021). In the research by Sırtmaç (2018), the geometric reasoning skills of eighth-grade students were examined on a different theoretical basis and the reasoning skills of the students were observed to be largely faulty/defective. Similarly, in Anıkaydın's (2017) research, it was stated that the geometric thinking levels of secondary school students were much lower than required. Bostancı et al. (2020) reported that the geometric reasoning skills of eighth-grade students were at a medium level. PISA and TIMSS exams can be given as examples of more comprehensive studies carried out in order to reveal a clearer picture of the geometric thinking skills of students in Türkiye. When the results of the PISA 2003, 2012 and 2022 exams, in which the mathematics learning area is predominant, are examined, it is seen that students generally perform lower in geometry than in other subject areas. According to the results of PISA-2003, more than 75% of the students in Turkey performed below the OECD average in the subject area of space and shape (geometry) (MoNE, 2005). When the PISA-2012 results are analyzed, it is seen that in the exam where the OECD average was 493, the students in Turkey remained below the average with 448 points; however, they obtained 448, 442, 447 and 443 points in the subject areas of i) change and relationships, ii) quantity, iii) uncertainty and data, iv) space and shape (geometry), respectively (MoNE, 2015). Based on these data, it can be said that the subject areas in which students are most unsuccessful are quantity and space and shape (geometry). According to the last PISA results in 2022, students in Turkey scored 453 points, which is below the OECD average (472), but showed the lowest performance in geometry among the different subject areas. The average scores obtained in the areas of change and relationships, quantity, uncertainty and data, space and shape (geometry) were 449, 455, 458 and 442, respectively (MoNE, 2022). In geometry, where the OECD average was 471, our students scored about 30 points below the average and showed the lowest performance in this subject area. In addition, the results of TIMSS can be analyzed as another international study. TIMSS (Trends in International Mathematics and Science Study), a project of the International Association for the Evaluation of Educational Achievement (IEA), is a survey to assess students' knowledge and skills in mathematics and science. According to the 2019 TIMSS results, Turkey performed significantly lower than the average, especially in geometry. Although there was a 27-point increase in geometry scores compared to the previous TIMSS results, this increase was insufficient to achieve average mathematics achievement. As a result, it was determined that algebra and geometry were the most unsuccessful learning areas for the eighth-grade sample of students in Turkey (MoNE, 2020). Therefore, the results of different studies in the literature and the results of this research generally indicate low or medium levels of geometric reasoning skills of students in our country. This research was conducted during the pandemic and most of the gains were provided in online courses. During this period, some students had difficulty accessing courses and therefore could not learn some subjects at all. In this context, the results of this research can also be interpreted in relation to this. However, another reason for the low level of students' geometric reasoning skills can be that the gains supporting the geometric reasoning process and the time allocated to these gains are limited in the secondary school curriculum. The step of the transition to theoretical reasoning takes place mainly at the secondary education level, both in our country and in other countries (Karpuz, 2018; MoNE, 2013; NCTM, 2000), and students are introduced to formal proof at the secondary education level. The limited number of gains regarding geometric reasoning at the secondary school level are given mainly in the second semester of the eighth-grade at the time of the High School Entrance Examination (LGS). During this period, a greater amount of time is generally devoted to solving exam questions rather than mathematical proof or explanation processes in the courses. In addition, geometry courses conducted in our country are not seen to teach proofs using colloquial language. The situations expressed in this part of the discussion can be stated to constitute an explanatory basis for the results of the research. It is important for improving students' geometric reasoning skill levels to create environments in which students can express their own thoughts. In addition, different activities and studies that will support students' perceptual and cognitive reasoning processes in relation to each other can be included in mathematics and +mathematics applications courses.

According to the results of the correlation analysis conducted within the scope of this research on the existence of the relationship between students' geometric reasoning skills and geometric reasoning self-efficacy perceptions, a positive, moderate, and significant relationship was found between said variables. These results can be associated with similar concepts discussed in different studies in this part of the study. Çağırgan-Gülten and Soytürk (2013) examined the relationship between sixth-grade students' geometry self-efficacy and geometry achievement and concluded that these variables were positively related to each other. In the research of

Bostancı et al. (2020) and Yenilmez and Korkmaz (2013), a significant but weak relationship between geometry self-efficacy perceptions and geometric reasoning skills of eighth-grade students. Unlike these, no significant relationships were found in Anıkaydın's (2017) research between middle school students' geometry self-efficacy perceptions and geometric thinking levels. The results of the Kinsey, Towle, O'Brien, and Bauer (2008) research conducted on this subject in the international literature can be examined. In the relevant research, a significant positive relationship was stated between the concept of self-efficacy and spatial ability, a determining factor of geometry performance. Similarly, Sudihartinih (2019) revealed that there is a moderate relationship between students' geometric thinking levels and their self-efficacy towards geometry. Therefore, despite the differentiation of the results of the research, meaningful relationships were generally seen between students' self-efficacy perceptions and geometry skills, but at different levels. In this context, the results in the literature are largely consistent with the results obtained from this research. Depending on the aforementioned relationship, students' self-efficacy perceptions can be improved in order to improve their geometric reasoning skills. It may be encouraged that academicians in teacher training institutions talk about the effects of affective components on learning with scientific justifications in their courses and share the results of different research with their students. It is thought that candidate teachers will be more conscious and willing to design learning environments that improve affective factors such as self-efficacy perception, and this will lead to very positive results in favor of the student. In addition, undergraduate-level courses can be provided to prospective mathematics teachers on the geometric reasoning process, its approaches, its importance, and how geometric reasoning can be improved.

Since this research is correlational research, it is limited to situation assessment. Causal studies can be conducted addressing students' geometric reasoning self-efficacy perceptions and geometric reasoning skill levels, and the reasons for student performances can be studied in more detail in such researches. Activities or learning environments can be designed to improve students' geometric reasoning skills, and the effects of activities/different learning environments on geometric thinking processes can be investigated through quasi-experimental research. In addition, scientific studies to be conducted at the secondary education level are considered to contribute greatly to the literature on the subject.

References

Akgül, A., & Çevik, O. (2003). *İstatistiksel analiz teknikleri [Statistical analysis techniques].* Ankara: Emek Ofset.

Alaylı, F. (2012). *Geometride şekil oluşturma ve şekli parçalarına ayırma çalışmalarında ilköğretim 6. 7. ve 8. sınıf öğrencilerinin düşünme süreçlerinin incelenmesi ve bu süreçteki düzeylerinin belirlenmesi [The investigation of thinking process of primary 6th ,7th and 8th grade students in the studies of composing and decomposing shapes in geometry and determination of their levels in this process]* (Unpublished doctoral thesis). Dokuz Eylül University, İzmir.

Alkan, H., & Taşdan, B. T. (2011). Farklı sınıf düzeylerindeki matematik öğretmen adaylarının gözünden matematiksel düşünme [Mathematical thinking through the eyes of prospective mathematics teachers at different grade levels]. *İnönü Üniversitesi Eğitim Fakültesi Dergisi [Journal of Inonu University Faculty of Education]*, *12* (2), 107-137.

Anıkaydın, Ö. (2017). *Öğrencilerin geometriye yönelik öz-yeterlik algıları, geometri tutumları ve geometrik düşünme düzeyleri arasındaki ilişkinin incelenmesi [Students oriented geometry self- efficacy beliefs, attitudes and geometry investigation of relationship* *between levels of geometric thinking]* (Unpublished master's thesis). Adnan Menderes University, Aydın.

Anıkaydın, Ö., & Elitok Kesici, A. (2017). Öğrencilerin geometriye yönelik öz-yeterlik algıları, geometri tutumları ve geometrik düşünme düzeyleri arasındaki ilişkinin incelenmesi [Students oriented geometry self- efficacy beliefs, attitudes and geometry investigation of relationship between levels of geometric thinking]. In A. Doğanay & O. Kutlu (Ed.), *Eğitim bilimleri araştırmaları II [Educational sciences research]* (pp. 9-28). Ankara: Akademisyen Kitabevi.

Ball, D., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick., G. Martin., & D. Schifter (Eds.), *A research companion to the principles and standards for school mathematics* (pp. 27-45). Reston, VA: National Council of Teachers of Mathematics.

Battista, M. T. (2007). The development of geometric and spatial thinking. In *Second handbook of research on mathematics teaching and learning* (pp. 843908). New York: Information Age.

Berkant, H. G., & Çadırlı, G. (2019). Ortaokul öğrencilerinin geometri öz yeterlik inançlarının ve geometrik düşünme becerilerinin incelenmesi [An analysis of secondary school students' geometry self-efficacy beliefs and their geometric thinking skills]. *Turkish Journal of Educational Studies, 6*(3), 29-52. https://doi.org/10.33907/turkjes.602382

Biber, M. (2012). *Duyuşsal özelliklerin probleme dayalı öğrenme sürecinde öğrencilerin matematiksel kazanımlara etkisi [The influence of affective factors on students' mathematical gains in the process of problem based learning]* (Unpublished doctoral thesis). Dokuz Eylül University, İzmir.

Bostancı, Ü. Y., Kuzu, O., & Sıvacı, S. Y. (2020). Sekizinci sınıf öğrencilerinin geometriye yönelik öz-yeterlik algıları ve geometrik akıl yürütme becerilerinin incelenmesi [Investigation of eighth-grade students' geometry self-efficacy perceptions and geometric reasoning]. *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi [Mehmet Akif Ersoy University Journal of Faculty of Education], 54*, 282-310. https://doi.org/10.21764/maeuefd.632387

Büyüköztürk, Ş. (2005). *Sosyal bilimler için veri analizi el kitabı [Handbook of data analysis for social sciences]* (5th ed.). Ankara: Cantekin Matbaası.

Büyüköztürk, Ş., Kılıç Çakmak, E., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F. (2018). *Bilimsel araştırma yöntemleri [Scientific research methods]* (24th ed.). Ankara: Pegem Akademi.

Cantürk-Günhan, B., & Başer, N. (2007). Geometriye yönelik öz-yeterlik ölçeğinin geliştirilmesi [The development of self-efficacy scale toward geometry]. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi [Hacettepe University Journal of Faculty of Education], 33* (33), 68-76.

Çağırgan-Gülten, D., & Soytürk, İ. (2013). İlköğretim 6. sınıf öğrencilerinin geometri öz yeterliklerinin akademik başarı not ortalamaları ile ilişkisi [The relation between 6th grade elementary school students' self-efficacy beliefs and academic achievement in geometry]. *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi [Mehmet Akif Ersoy University Journal of Faculty of Education], 25,* 55-70.

Clements, D. H. (2003). Teaching and learning geometry. https://www.researchgate.net/profile/Douglas_Clements/publication/258933229_Teaching_and_learning_geometry/links/557dd19508aeea18b777c211.pdf

Damaryanti, D., Mariani, S., & Mulyono, M. (2017). The analysis of geometrical reasoning ability viewed from self-efficacy on connected mathematic project (CMP) learning etnomathematics-based. *Unnes Journal of Mathematics Education, 6*(3), 325-332. https://doi.org/10.15294/ujme.v6i3.17126

De Vellis, R. F. (2003). *Scale development theory and applications* (2nd ed.). SAGE Publication, Inc.

Douek, N. (2005). Communication in the mathematics classroom: argumentation and development of mathematical knowledge. In A. Chronaki, I. M. Christiansen (Eds.), *Challenging perspectives on mathematics classroom communication* (pp. 145-172). Greenwich, CT: Information Aga Publishing.

Duval, R. (1995). Geometrical Pictures: Kinds of representation and specific processings. In R. Sutherland & J. Mason (Ed.), *Exploiting mental imagery with computers in mathematics education* (pp. 142-157). Heidelberg: Springer Berlin.

Duval, R. (1998). Geometry from a cognitive point a view. In C. Mammana & V. Villani (Ed.), *Perspectives on the Teaching of geometry for the 21st century* (s. 37-52). Dordrecht: Kluwer Academic Publishers.

Erkek, Ö., & Işıksal Bostan, M. (2015). Uzamsal kaygı, geometri öz-yeterlik algısı ve cinsiyet değişkenlerinin geometri başarısını yordamadaki roller [The role of spatial anxiety, geometry self-efficacy and gender in predicting geometry achievement]. *İlköğretim Online [Elementary Education Online]*, *14*(1), 164-180. https://doi.org/10.17051/io.2015.18256

Fidan, Y., & Türnüklü, E. (2010). İlköğretim 5. sınıf öğrencilerinin geometrik düşünme düzeylerinin bazı değişkenler açısından incelenmesi [Examination of 5th grade students' levels of geometric thinking in terms of some variables]. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi [Pamukkale University Journal of Faculty of Education]*, *27*(27), 185-197.

Fischbein, E., & Nachlieli, T. (1998). Concepts and figures in geometrical reasoning. *International Journal of Science Education, 20*(10), 1193-1211. https://doi.org/10.1080/0950069980201003

Gündoğdu, S. (2013). *7. ve 8. sınıf öğrencilerinin sahip olduğu matematiksel güç ile matematik öz yeterliği arasındaki ilişki [Relationship between 7 th and 8th grade students' mathematical power and mathematics self-efficacy]* (Unpublished master's thesis). Osmangazi University, Eskişehir.

Güven, B., & Karpuz, Y. (2016). Geometrik muhakeme: bilişsel perspektifler [Geometric reasoning: cognitive perspectives]. In S. Arslan, E. Bingölbali & İ. Zembat (Ed.), *Matematik eğitimde teoriler [Theories in mathematics education]* (pp. 245-263). Ankara: Pegem Akademi.

Güzeller, G. (2018). *5 ve 6. sınıf öğrencilerinin şekilsel kavram teorisi çerçevesinde temel geometrik kavramları anlamlandırmasının incelenmesi [Exploring 5th and 6th grade students' understanding of basic geometrical concepts based on the theory of figural concept]* (Unpublished master's thesis). Hacettepe University, Ankara.

Hutcheson, G. D., & Sofroniou, N. (1999). *The Multivariate Social Scientist: Introductory statistics using generalized linear models.* London: Sage Publications.

İlhan, A. (2019). *İlköğretim matematik öğretmen adaylarının görsel matematik okuryazarlık algı düzeyleri ile geometri konusunda akıl yürütme beceri ve performans düzeyleri arasındaki ilişkinin incelenmesi [Investigation of the relationship of elementary mathematics teacher candidates' between visual mathematics literacy perception level, reasoning skill on geometry and performance levels]* (Unpublished doctoral thesis). İnönü University, Malatya.

Jones, K. (1998). Theoretical frameworks for the learning of geometrical reasoning. *Proceedings of the British Society for Research into Learning Mathematics, 18*(1-2), 29-34.

Jöreskog, K. G., & Sörbom, D. (1996). *LISREL 8: User's reference guide*. Chicago: Scientific Software International.

Kaba, Y., Boğazlıyan, D., & Daymaz, B. (2016). Ortaokul öğrencilerinin geometriye yönelik tutumları ve özyeterlikleri [Ortaokul öğrencilerinin geometriye yönelik tutumları ve özyeterlikleri]. *The Journal of Academic Social Science Studies, 52*, 335-350. https://doi.org/10.9761/JASSS3727

Kan, A. (2009). Ölçme sonuçları üzerinde istatistiksel işlemler [Statistical operations on measurement results]. In H. Atılgan (Ed.), *Eğitimde ölçme ve değerlendirme [Measurement and evaluation in education]* (pp. 397–456). Ankara: Anı Yayıncılık.

Karakarçayıldız, R. Ü. (2016). *7. sınıf öğrencilerinin geometrik düşünme düzeyleri ile çokgenleri sınıflama becerileri arasındaki ilişki [Geometrical thinking levels and polygons classification skills of 7 th grade students and relationship between them]* (Unpublished master's thesis). Eskişehir Osman Gazi University, Eskişehir.

Karasar, N. (2004). *Bilimsel araştırma yöntemi [Scientific research method]* (14^{th} ed.). Ankara: Nobel Yayıncılık.

Karpuz, Y. (2018). *Duval'in bilişsel modeline uygun tasarlanan öğrenme ortamının değerlendirilmesi [The evaluation of the learning environment designed in accordance with Duval's cognitive model]* (Unpublished doctoral thesis). Karadeniz Technical University, Trabzon.

Karpuz, Y., Koparan, T., & Güven, B. (2014). Geometride öğrencilerin şekil ve kavram bilgisi kullanımı [Using figure and concept knowledge in geometry]. *Türk Bilgisayar ve Matematik Eğitimi Dergisi [Turkish Journal of Computer and Mathematics Education], 5*(2), 108-118. https://doi.org/10.16949/turcomat.16832

Kaur, B., & Lam, T. T. (2012). Reasoning, communication and connections in mathematics: An introduction. In B. Kaur & T. T. Lam (Eds.), *Reasoning, communication, and connections in mathematics: Yearbook 2012 association of mathematics educators* (pp. 1–10). Singapore: World Scientific Publishing.

Kılıç, S. (2013). Örnekleme yöntemleri [Sampling methods]. *Journal of Mood Disorder, 3*(1), 44-46. https://doi.org/10.5455/jmood.20130325011730

Kızıltoprak, A. (2020). *Ortaokul öğrencilerinin dörtgenlere ilişkin geometrik muhakemelerinin gelişmesi [Development of middle school students' geometric reasoning about quadrilaterals]* (Unpublished doctoral thesis). Anadolu University, Eskişehir.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds). (2001). *Adding it up: helping children learn mathematics.* Washington, D.C.: National Academies Press.

Kinsey, B. L., Towle, E., O'Brien, E. J., & Bauer, C. F. (2008). Analysis of self-efficacy and ability related to spatial tasks and the effect on retention for students in engineering. *The International Journal of Engineering Education, 24*(3), 488-494.

Köse, N., Tanışlı, D., Özdemir-Erdoğan, E., & Yüzügüllü-Ada, T. (2012). İlköğretim matematik öğretmen adaylarının teknoloji destekli geometri dersindeki geometrik oluşum edinimleri [Elementary mathematics teacher candidates' acquisition of geometric constructions in technology-assisted geometry course]. *Mersin Üniversitesi Eğitim Fakültesi Dergisi [Mersin University Journal of Faculty of Education], 8*(3), 102-121.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: interaction in classroom cultures* (pp. 229-270). Hillsdale, NJ: Laurence Erlbaum.

Michael – Chrysanthou, P., & Gagatsis, A. (2013). Geometrical figures in geometrical task solving: an obstacle or a heuristic tool? *Acta Didactica Universitatis Comenianae – Mathematics, 13*, 17-30.

Michael, P. (2013). *Geometrical figure apprehension: cognitive processes and structure* (Unpublished doctoral thesis). The University of Cyprus, Cyprus.

Michael, P., Gagatsis, A., Avgerinos, E., & Kuzniak, A. (2011). Middle and high school students' operative apprehension of geometrical figures. *Acta Didactica Universitatis Comenianae–Mathematics, 11*, 45- 55.

Ministry of National Education [MoNE] (2005). *PISA 2003 projesi ulusal nihai rapor[PISA 2003 national final report]*. Ankara: MEB Yayınları. Retrieved from https://pisa.meb.gov.tr/eski%20dosyalar/wp-content/uploads/2013/07/PISA-2003-Ulusal-Nihai-Rapor.pdf

Ministry of National Education [MoNE] (2013). *Ortaokul matematik dersi öğretim programı [Secondary school mathematics curriculum]*. Ankara: MEB Yayınları. Retrieved from https://matematikhayat.wordpress.com/wp-content/uploads/2012/09/matematik_5-8.pdf

Ministry of National Education [MoNE] (2015). *PISA 2012 araştırması ulusal nihai rapor[PISA 2012 national final report]*. Ankara: MEB Yayınları. Retrieved from https://pisa.meb.gov.tr/meb_iys_dosyalar/2020_12/23172540_pisa2012-ulusal-nihai-raporu.pdf

Ministry of National Education [MoNE] (2020). *TIMSS 2019 Türkiye ön raporu [TIMSS 2019 Türkiye preliminary report]*. Ankara: MEB Yayınları. Retrieved from https://odsgm.meb.gov.tr/meb_iys_dosyalar/2020_12/10175514_TIMSS_2019_Turkiye_On_Raporu_.pdf

Ministry of National Education [MoNE] (2022). *PISA 2022 Türkiye raporu [PISA 2022 Türkiye report]*. Ankara: MEB Yayınları. Retrieved from https://pisa.meb.gov.tr/meb_iys_dosyalar/2024_03/21120745_26152640_pisa2022_rapor.pdf

Morsanyi, K., Prado, J., & Richland, L. E. (2018). The role of reasoning in mathematical thinking. *Thinking & Reasoning*, *24*(2), 129-137. https://doi.org/10.1080/13546783.2018.1435425

Mutluoğlu, A. (2019). *6. sınıf matematik dersi geometri ve ölçme öğrenme alanında geliştirilen bir sanal manipülatif takımının (MATMAP) öğrencilerin akademik başarılarına, geometriye yönelik tutumlarına ve geometrik muhakeme süreçlerine etkisi [The effects of virtual manipulatives, developed for 6th grade mathematics lesson in geometry and measurement learning area, on students' academic achievement, attitudes towards geometry, and geometrical reasoning processes]* (Unpublished doctoral thesis). Necmettin Erbakan University, Konya.

Mutluoğlu, A., & Erdoğan, A. (2021). 6. sınıf öğrencilerinin dörtgenler hakkındaki geometrik muhakeme süreçleri [6th grade students' geometrical reasoning processes about quadrilaterals]. *OPUS Uluslararası Toplum Araştırmaları Dergisi [International Journal of Society Researches]*, *16*(27), 236-265. https://doi.org/10.26466/opus.673833

NCTM (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

OECD. (2023). *PISA 2022 Assessment and analytical framework.* Paris: OECD Publishing. https://doi.org/10.1787/dfe0bf9c-en

Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary Educational Psychology*, *24*(2), 124-139. https://doi.org/10.1006/ceps.1998.0991

Pietsch, J., Walker, R., & Chapman, E. (2003). The relationship among self-concept, self-efficacy, and performance in mathematics during secondary school. *Journal of Educational Psychology*, *95*(3), 589–603. https://doi.org/10.1037/0022-0663.95.3.589

Sabirin, M., Aminah, S., & Atsnan, M. F. (2021). Students' skills performing mathematical reasoning and communication through group ınvestigation learning model. *Jurnal Pengembangan Pembelajaran Matematika*, *3*(2), 112-128.

Sarpkaya Aktaş, G. (2020). Etkili matematik öğretimi ile oluşturulan beceriler [Skills created by effective mathematics teaching]. In M. Ünlü (Ed.), *Uygulama örnekleriyle matematik öğretiminde yeni yaklaşımlar [New approaches in mathematics teaching with application examples]* (pp. 67-91). Ankara: Pegem Akademi.

Seçer, İ. (2015). *SPSS ve LISREL ile pratik veri analizi analiz ve raporlaştırma [Practical data analysis analysis and reporting with SPSS and LISREL].* Ankara: Anı Yayıncılık.

Senk, S. L. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education*, *20*(3), 309-321.

Sırtmaç, G. (2018). *Sekizinci sınıf öğrencilerinin geometrik akıl yürütme becerilerinin incelenmesi [An investigation of geometric reasoning skills of eighth grade students]* (Unpublished master's thesis). Dokuz Eylül University, İzmir.

Sudihartinih, E., & Wahyudin. (2019). Analysis of students' self-efficacy reviewed by geometric thinking levels and gender using Rasch model. *Journal of Engineering Science and Technology, 14*(1), 509 – 519.

Sumarsih, S., Budiyono, B., & Indriati, D. (2018). Profile of mathematical reasoning ability of 8th grade students seen from communicational ability, basic skills, connection, and logical thinking. *Journal of Physics: Conference Series*, *1008*(1). https://doi.org/10.1088/1742-6596/1008/1/012078

Şimşek, Ö. F. (2007). *Yapısal eşitlik modellemesine giriş- temel ilkeler ve LISRELL uygulamaları [Introduction to structural equation modeling - basic principles and LISRELL applications.]*. İstanbul: Ekinoks Yayınları.

Tabachnick, B. G., & Fidell, L. S. (2007). *Using multivariate statistics*. New York: Allyn and Bacon.

Tavşancıl, E. (2005). *Tutumların ölçülmesi ve SPSS ile veri analizi [Measurement of attitudes and data analysis with SPSS].* Ankara: Nobel Yayıncılık.

Tutan, S. (2019). *Geometrik muhakeme süreçleri bağlamında ortaokul matematik öğretmenlerinin geometri içerikli derslerinin incelenmesi [The examination of middle school mathematics teachers' geometry-based courses in the context of geometric reasoning processes]* (Unpublished master's thesis). Gaziantep University, Gaziantep.

Umay, A. (2003). Matematiksel muhakeme yeteneği [Mathematical reasoning ability]. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi [Hacettepe University Journal of Faculty of Education],* 24 (24), 235-243.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry (Final report of the cognitive development and achievement in secondary school geometry project).* Chicago: University of Chicago, Department of Education.

Ünlü, M. (2014). *Geometri başarısını etkileyen faktörler: Yapısal eşitlik modellemesi [Factors Affecting Geometry Success: A Structural Equation Modelling]* (Unpublished doctoral thesis). Necmettin Erbakan University, Konya.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2016). *İlkokul ve ortaokul matematiği: Gelişimsel yaklaşımla öğretim [Elementary and middle school mathematics: Teaching with a developmental approach].* (Trans. Ed.: S. Durmuş). Ankara: Nobel yayıncılık.

Yavuz Mumcu, H. (2019). İlköğretim matematik öğretmen adaylarının matematiksel muhakeme öz-yeterlik inançlarının incelenmesi: Bir ölçek geliştirme ve uygulama çalışması [Investigation of mathematical reasoning self-efficacy beliefs of pre-service elementary mathematics teachers: A scale development and implementation study]. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi [Ahi Evran University Journal of Kırşehir Faculty of Education]*, *20* (3), 1239-1280. https://doi.org/10.29299/kefad.2019.20.03.007

Yenilmez, K., & Korkmaz, D. (2013). Relationship between 6th, 7th and 8th grade students' self-efficacy towards geometry and their geometric thinking levels. *Necatibey Faculty* *of Education Electronic Journal of Science and Mathematics Education, 7*(2), 268-283. https://doi.org/10.12973/nefmed210

Zimmerman, B. J. (1995). Self-efficacy and educational development. A. Bandura (Ed.), In *Self-efficacy in changing societies* (pp. 202-231). Cambridge, UK: Cambridge University Press.

Declarations

Acknowledgementa: *This research was produced from the master's thesis titled 'Examining the relationship between geometric reasoning skills and self-efficacy perceptions of 8th grade students' conducted by the first author under the supervision of Assoc. Prof. Dr. Hayal Yavuz Mumcu.*

Funding: *No funding source is reported for this study.*

Ethics Statements: *Author/s declare presence of Ethics Statements that needed for ethical conduct of research using human subjects.*

Conflict of Interest: *No potential conflict of interest was reported by the author(s)*

Data availability: *The data that support the findings of this study are available from the corresponding author upon reasonable request.*