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How Do Assimilation and Accommodation Occur in The Translation Process of Representation?

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This study aims to describe the translation process of representation in mathematics education students' solving of mathematical problems in the form of graphs. The translation process involves four activities: unpacking the source, preliminary coordination, constructing the target, and determining equivalence. The study was conducted on 65 students who took Calculus at three different universities in East Java Province, Indonesia. Research data in the form of answers to mathematical problems, video recordings, and interviews were analyzed based on the activity of the translation process within the accommodation and assimilation framework. Based on data analysis, the characteristics of the representation translation process are obtained in three categories, namely the symbolic-algebraic translation process (SA), the verbal translation process (V), and the symbolic-conceptual translation process (SC). When "unpacking the source" and "preliminary coordination," SA looks difficult, so it changes the equations and graphs for completion several times. V verbally smoothly performs four translational process activities. However, subject V has doubts about the graph made after reading the question back. SC uses graph equations until it finds a solution in the form of a graph. However, after reflection, SC resolves the problem with the theory of monotony. It is important for the future teacher to understand the translation process of representation, especially given the difficulty students have solving mathematical problems. Prospective teachers are

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expected to be able to develop meaningful learning with various forms of representation so that students can connect their concepts to problem solving.

Introduction

One of the most important mathematical abilities possessed by students is representation. This is supported by NCTM (2000) which states that representation is one of the standards in learning mathematics. Representation becomes a tool to improve understanding in mathematics learning (Ainsworth, 1999; Bal, 2015; Gagatsis et al., 2004; Hiebert & Carpenter, 1992; Kaput, 1989; Porzio, 1994; Skemp, 1987), building strong mathematical connections (Adu-Gyamfi et al., 2016; Gagatsis et al., 2004; Mhlolo et al., 2012; Shiakalli & Gagatsis, 2006), as well as solving mathematical problems (Jao, 2013; Pape, 2004).

The ability to relate various forms of representation can be seen from the translation between the forms of representation. The process of translation of various forms of representation is a cognitive process that connects a form of representation (source) to a different form of representation (target), without changing the notated meaning (Adu-Gyamfi et al., 2012; Ainsworth, 1999; Bossé et al., 2011; Duval, 1999, 2006). The representation translation process helps students understand concepts, solve problems, and as an indicator of learning success (Bal, 2014, 2015; Biber, 2014; Bossé et al., 2014a; Çelik & Saglam-Arslan, 2012; Dündar, 2015; Rahmawati, 2019; Rahmawati & Anwar, 2020). Therefore, the translation between various forms of representation plays an important role in learning mathematics. The importance of the ability to translate representations, cannot escape the role of teachers/educators in facilitating students in providing experiences regarding various forms of representation. However, there are still many students, prospective teachers and teachers who have limited ability to translate representations. The limited ability to translate various forms of representation will reduce the success of solving mathematical problems (Villegas et al., 2009). The limitations of the representational translation abilities of students, teacher candidates, and teachers, especially in the form of graphs, have been shown in several previous studies. Delice & Sevimli (2010) found that prospective teachers only want to use algebraic representations because they are accustomed to using algebraic forms of representation. İpek & Okumuş (2012) obtained those 48 teacher candidates used more verbal representations than graphic representations and numerical algebra in the problem solving process. In a preliminary study by the researcher, it was shown that more students succeeded in solving symbolic form problems than verbal forms and diagrams (Swastika et al., 2018). In understanding the difficulties of students who have limited ability to translate graphic representations, educators need to know the details of the translation process between graphic representations, namely the translation process of graphic representations. Therefore, mathematics education students as prospective teachers must know how the process of students translating between representations works.

In the research of Bossé et al. (2014), a ranslation framework of four translational activities between mathematical representations was shown. However, this study did not examine the characteristics of the translation process of mathematical representations. This study describes the characteristics of the translation process that is photographed using assimilation and accommodation. Swastika, et al. (2020) show a phenomenon related to the translation process of the representation of graph f' to graph f on one of the subjects (S2). S2 has difficulty when using symbolic representation completion, especially when the initial coordination activity is thus unable to maintain the equivalence of the source and target representations. This shows that students have not been able to confirm components in the source representation in the form of graphs using symbolic representations (Bossé et al., 2011). Whereas the purpose of translational representation is to maintain the equivalence of the components of the source representation with the components of the target representation (Lesh et al., 1987). This confirms the findings of Dreyfus dan Eisenberg (Adu-Gyamfi et al., 2012) and Rahmawati, et al. (2017) that students fail or are not optimal in confirming the equivalence of the component representation of the source and the target component even though they can translate from the source representation to the target representation in the form of a function graph. However, this study did not examine the characteristics of the mathematical translation process.

Theranslationn process of students in solving mathematical problems is an activity that occurs in the human brain. The incoming information and data are processed in the brain, so that what is already inside needs adjustments, even changes. In the preliminary study, the adjustment process occurs when the components of the source graph are adjusted with the concept they have into an intermediate representation. The process of processing information and data on the schema can be done in two ways, namely assimilation and accommodation. Piaget (in Subanji, 2007) mentions assimilation and accommodation as two individual processes in adapting to the environment.

In this study, assimilation is a process of directly interpreting the problem of translation of representations by using the schema they have, while accommodation is interpreting the problem of representation through the process of changing the existing schema or forming a new schema to match the structure of the problem. In assimilation, the subject directly interprets and receives a new stimulus with the schema he has. In accommodation, the subject cannot immediately understand the new stimulus because the structure of the knowledge he has is not in accordance with the structure of the problem, so it is necessary to change or add to the knowledge scheme he has. The stimulus in question is a mathematical object in the form of facts, concepts, principles related to the translation of the representation of mathematical problems. In this study, the translational process of graph representation of mathematical problems was analyzed using four translational activities (Bossé et al., 2014) and the framework of assimilation and accommodation because there is a relationship between scheme, assimilation, and accommodation in the translation process of representation. The mathematical translation activities consist of activities of unpacking the source, preliminary coordination, constructing the target, and determining equivalence (Bossé et al., 2014).

Method

The type of research used in this research is qualitative research. Qualitative research intends to understand phenomena about what is experienced by research subjects such as behavior, perception, motivation, and action holistically, by describing it in the form of words and language, in a special natural context and by utilizing various scientific methods (Creswell, 2015). In this study, researchers tried to conduct a thorough, careful, and in-depth examination of students regarding what was thought of the assimilation and accommodation framework to obtain an idea of the translation process of student representation in solving a given problem.

Participants

The research subjects taken are mathematics education students who have or are taking Calculus 1. The selection of this subject was made because the subject already knew the concept of function graphs in the lecture. The demographics of the research subjects are shown in Table 1.

Table 1. Demographics of Research Subjects

Region	Number of Students
State University in Malang	10
Private University in Malang	30
Private University in Blitar	25

Data Collection

Prospective research subjects were given a representation translation task (Figure 1) and asked to think aloud when working on a representation translation task. Furthermore, the subject was interviewed to reveal further the process of student representation and to clarify the results of their work. So that no information is missed, and the data obtained is guaranteed to be valid, the interview is recorded using a camera or mobile phone that produces oral answers in the form of recordings. The results of the interview recordings are transcribed in detail and combined with field notes and the results of researchers' observations. The results of the interview recordings will be transcribed in detail which will be combined with field notes and the results of the researchers' observations during the interview. The triangulation carried out in this study is data triangulation which is carried out by comparing or checking think aloud data, the work of prospective subjects and interviews to strengthen research evidence (Creswell, 2015). From the data obtained from the student answer sheets, think aloud, and interviews, subjects were grouped based on the form of representation used in the translation process of problem solving.

Graph of f' is represented below. If f continious at $\mathbb{R}$ by $f(0) = 0$, sketch graph of f.

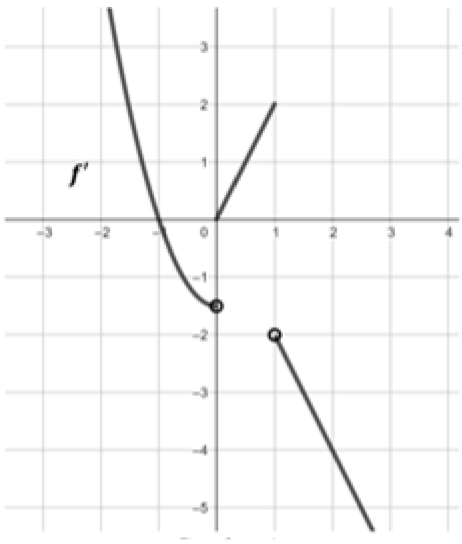

Figure 1. Representation Translation Task

Data Analysis

Researchers conduct data analysis with stages of data reduction, data presentation, and drawing conclusions. The data from "think aloud" and the results of student work and interviews are summarized carefully and in detail. Next, the data are presented by conducting an assembly, organizing data from successfully collected information for the withdrawal of conclusions and the establishment of subsequent activities. Drawing conclusions is carried out based on the results of the analysis of data that has been collected, both in the form of written data and recorded results. The conclusions drawn in this study are based on expressions in the form of visual representations (images, graphs, or tables), symbolic representations (mathematical statements, symbols of quadratic functions), and verbal representations (written text and words) based on the processes of assimilation and accommodation as in Table 2.

Table 2 Indicators of the Translation Process in Solving Mathematical Problems

Translation Activity	**Process**	**The description of Thinking Process Activities**	**Indicators**
Unpacking the Source	Describe what is known and what is asked and identify relevant information	Assimilation	Finding important elements in understanding the problem • Directly shows points (intersection, stationary point) on the graph • Directly shows the opening direction of the graph ● Directly shows graph up and down intervals

Translation Activity	**Process**	**The description of Thinking Process Activities**	**Indicators**
		Accommodation	• Carry out a process (such as making a graphic example of a known equation/polynomial, rereading an existing problem, or so on) to further identify the degree of the graphic equation • Repeatedly reading the problem to determine the important elements of the problem
Preliminary coordination	Linking the information that has been unpacked at the stage of unpacking the source with the concept that have been understood	Accommodation	• Directly determine the general equation from the source graph • Directly relate general equations to the concept of derivative/integral polynomial equations • Directly relate the concept of the stationary point of the graph to the gradient of the tangent to the graph
	Prepare descriptions/features that may be used to construct the target representation	Accommodation	• Rethinking the concept of an up and down interval of the source graph graph with the position of the target graph on the X axis to further coordinate the relationship between the two • Rethinking the concept of a stationary point of the source graph with the position of the target graph on the X axis to further coordinate the relationship between the two
Constructing the target	Transfer the information/information contained in the source representation to the target representation,	Assimilation	• Determine directly the derivative/integral result of the general equation of the source graph • Directly determine the stationary points on the target chart • Sketch the target chart
	Complete the description on the target representation	Accommodation	• Thinking over and over to set the target chart position about the x axis (cut/offend sb x or above/below sb x) • Solve existing problems indirectly • Solve problems that are different from the problem-solving plan that has been made from scratch
Determining equivalence	Check the similarities between the source and target representations and consider similar ideas in the source and target representations	Assimilation	• Directly determining the similarity/conformity of the results of the settlement is a representation of the target.
		Accommodation	• Create new problem solving and are not sure of the correctness of the results that have been obtained • Carry out a process (such as creating tables, rereading existing problems, or so on) to determine how to re-examine the results obtained

Findings

As many as 59 (90.7%) of the 65 students gave complete answers accompanied by explanations, while 9 students gave answers without explanations so that the answers were not analyzed. Data obtained from student answer sheets, think-alouds, exercises, and interviews is grouped based on the form of representation used in the translation process of solving questions. Furthermore, 6 students were described as subjects by considering: (1) the form of representation used in problem solving, (2) the willingness of the subject to be investigated further and (3) the subject's ability to explain the problem-solving process carried out. From this grouping, three categories were obtained in the representation translation process which can be seen in Table 3 below.

Table 3. **Categorization of Findings of the Representation Translation Process**

Selected Subject Categorization	**Selected Subject Categorization**
Category I : Symbolic-Algebraic translation	S1
Category II : Verbal translation	S2
Category III: Symbolic-Concept Translation Process	S3

Category I : Symbolic-Algebraic Translation Process

Unpacking the Source

S1 identifies the graph of the function of f' in the problem. S1 is confused when observing the shape of the graph f'. This can be seen from the following statement S1.

S1 : From question number one, it is known here, (S1 is silent) so this is the graph of the function, which is the function f'. So later we will ... solve the curves one by one first, find the equation of the line.

The statement of S1 shows that S1 is experiencing obstacles. S1 attempts to resolve the confusion by assuming the graph f' consists of 3 different graphs (Figure 2).

Figure 2. Results of S1 Exploration in Observing Function Graphs f'

S1 interprets the components of the problem, where S1 undergoes a process of accommodation by interpreting the graphic form f' (source) as well as the conditions on the source. S1 attempts to find equations of some form in the graph f'. This corresponds to the S1 statement and the work in Figure 3 below.

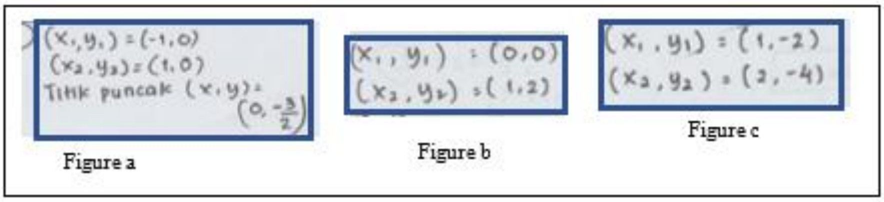

Figure 3. Results of S1 Exploration Identifying Points on Graph f'

Furthermore, S1 traces the related components in the source representation showing S1 also experiences accommodation. S1 looked back at the problem when asked by the researcher about information on the problem. S1 knows the information in the problem that $f(0) = 0$. Other information obtained by S1 on the problem is that there are boundaries (intervals) in each form of the graph of $f'(x)$. This shows the process of assimilation in S1 when looking back at the information that might be obtained on the problems.

Preliminary Coordination

S1 guesses the source relationship by trying to determine the equation of the function of each example graph 1, 2, and 3. S1's effort to determine the equation of the function using the equation of function shows that S1 is assimilated. This is in accordance with the following statement S1 anf Figure 4.

S1 : Find the equation of the line using the formula $y = a(x - x_1)(x - x_2)$*, for this vertex* $\left(0, -\frac{3}{2}\right)$*. So x and y are substituted* $-\frac{3}{2} = a \cdot (0 - (-1)) \cdot (0 - 1)$*. Then* $-\frac{3}{2} = a \cdot 1 \cdot (-1)$ *so* $a = \frac{3}{2}$*. Next, to find the equation, substitute a for* $y = \frac{3}{2} \cdot (x + 1) \cdot (x - 1)$ *then* $y = \frac{3}{2} \cdot$ *is removed as* $x^2 - x + x - 1$*. So* $y = \frac{3}{2} \cdot (x^2 - 1)$*. That is for his* y'

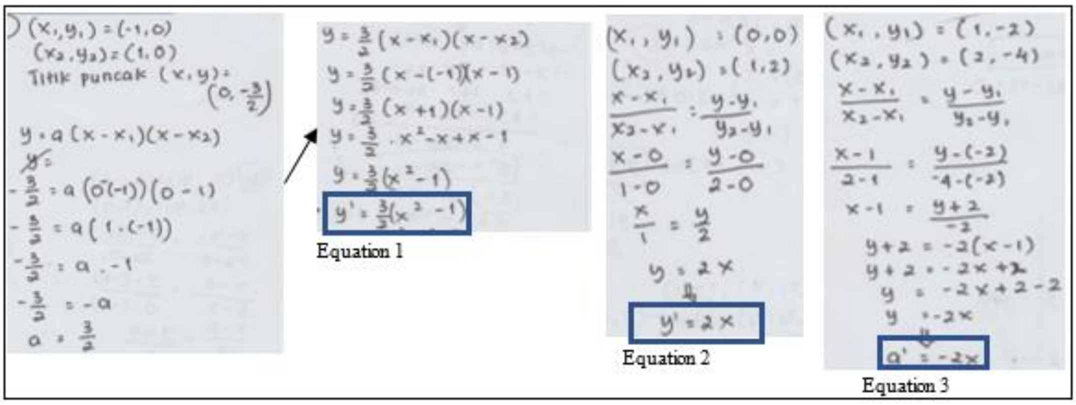

Figure 4. *Results of S1 Exploration Making Graphic Equations* f'

Constructing the Target

S1 builds and understands the components in the target. S1 undergoes an assimilation process when building the target component by performing an integral procedure to obtain the equation of the graph f and the graph of f (target). S1 directly performs the integral procedure. S1 obtains the integral result which is presented in Figure 5 as follows.

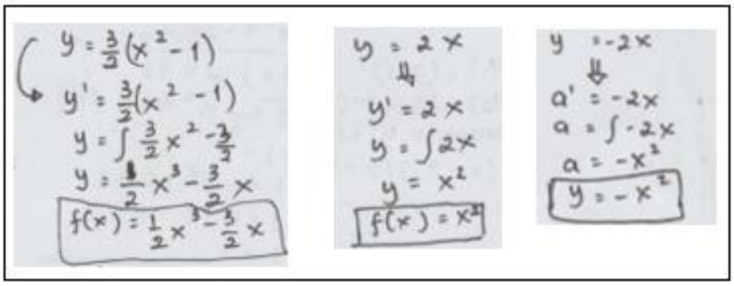

Figure 5. Results of S1 Exploration in Performing Integral

S1 did trial and error to find the points in the y equations of graphs 1, 2, and 3. This is in accordance with the results of S1's exploration in Figure 6 below.

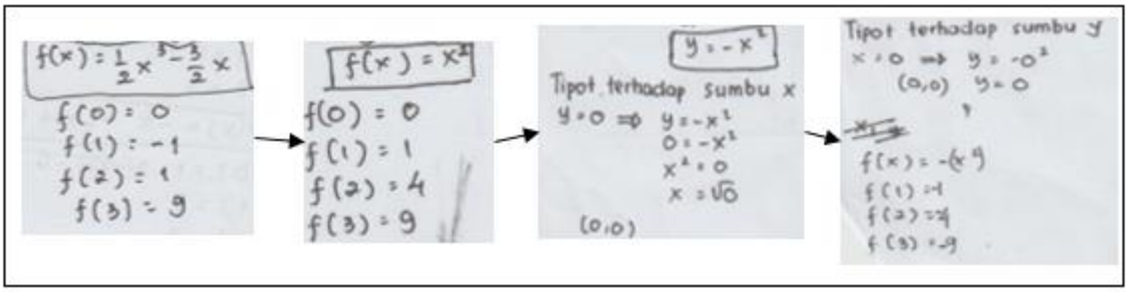

Figure 6. Results of S1 Exploration Determining the Graph Point f

The results of S1's work, which are in accordance with the S1 statement above, are presented in Figure 7, as follows.

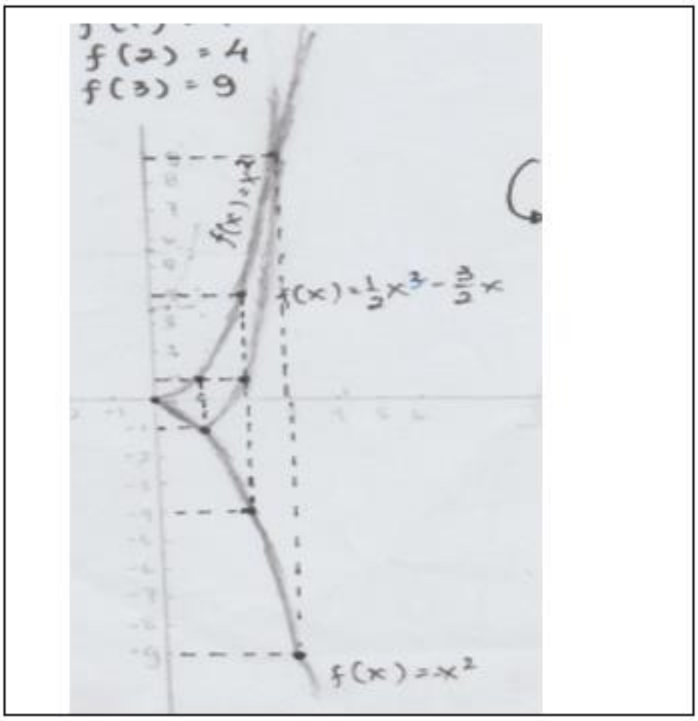

Figure 7. Results of S1 Answer Sketches

Determining Equivalence

S1 evaluates the suitability of source and target and improves the relationship of source and target. This is in accordance with the excerpt of an interview with S1.

P : Then from your work, is it correct? Have you checked?
S1 : Hmmm seems to have been (silence looking at the answer)
P : Certain?
S1 : Hmmm maybe something is wrong (silently observing the answers and questions) Oh yes.... when integrating not using c, hmm I'll fix it first (correct the answer)

S1 was confused when he looked back at the answers he had obtained (reflection). S1 realizes that the integral process used is not correct based on S1's statement, "Oh yes.... when integrating, don't use c". S1 corrected the answer he got and explained that the results of the integral process he did did not contain "c" (accommodation). Next, S1 remembers the concept of limit to determine "c" in the equation $y = f(x)$ in graphs 1, 2, and 3 (accommodation).

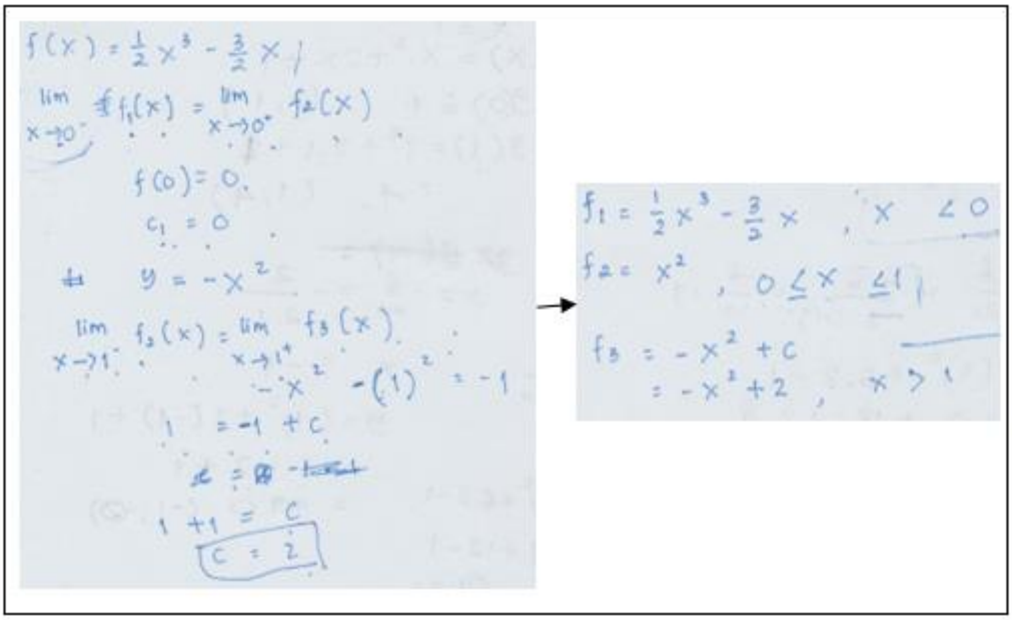

Figure 8. S1 Exploration Results Fix Troubleshooting

Having obtained a point that passes through the graph f, S1 draws a sketch of the graph f requested in the problem. The results of S1's work are shown in Figure 9 below.

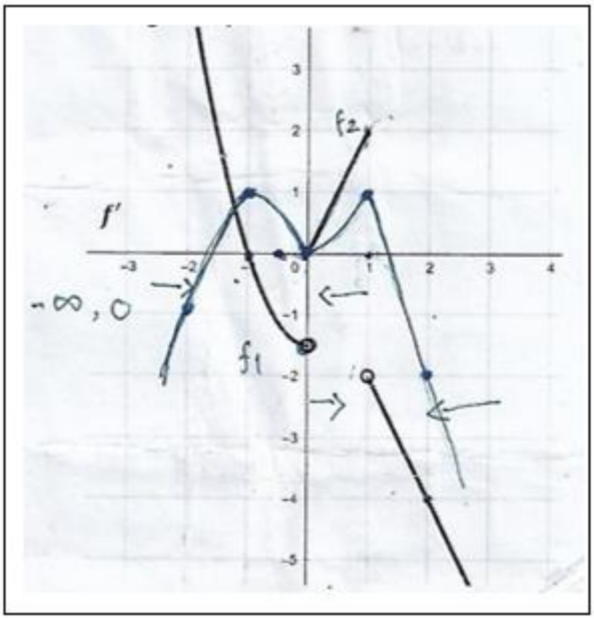

Figure 9. Results of the S1's Target Representation

Category II : Verbal Translation Process

Unpacking the Source

S2 identifies known elements of the problem. S2 reveals that there is a graph of f in the problem and is asked to sketch the graph of f. S2 unpacks the source by verbally expressing a representation of the problem. S2 interprets the components of the problem by revealing that there are graph intervals with certain properties at each interval. Furthermore, S2 tracks the corresponding component in the source representation. S2 mentions graphs as a quadratic equation and a linear equation. When S2 will reveal the properties of the graph f' at intervals of $(0,1)$, S2 pauses for a while and then reveals the properties of f' as a function of ascending. The silence of S2 indicates the occurrence of accommodation in the S2 thinking structure. This can be seen from the following S2 statement.

> *S2 : It is known that the function f with graph f' is presented in the figure, if f is a continuous function with f(0)=0, sketch the graph of f. It means that his f(x) is less than 0. (silently looking at the problem) Then if (while pointing at the graph at the point (0,0)) from the interval 0 to 1, this graph of f^' is an increasing function, if from 1 to no until it is a descending function. (pause for a few seconds looking at the problem) mmm... When f' eh.. when f=0 it's on the left it's 0 it's a descending function, but if the one to the right is 0 it's an up function. So it's possible that later on the f(x) graph it will be the inflection point. Likewise, f(1) is an inflection point.*

Preliminary Coordination

S2 predicts a source and target relationship. S2 verbally predicts the nature of the graph f' (source) using the concepts of monotony, gradient and concavity. S2 relates the known concepts of concavity and monotony to represent the elements on the graph. S2 also relates the concept of the turning point to be applied to the graph $f'(x)$ at the point (-1,0). This shows that S2 performs assimilation activity. This is in accordance with the following interview with S2.

S2 : *mmm... the first one is about eee... what... the curve... when the curve goes up, the function goes up, the curve is said to be ee... its first derivative, the function will increase when f' eh when the f(x) is more than 0, so the function goes up and down, continues from the concavity of the curve, concave up and concave down, when it is shaped like this, the shape of the curve (while pointing to the curve in figure 1 in the problem) is like that.*

I : *From that question, could there be hidden information that you can get?*

S2 : *mmm... mostly here I think if eee... this point (-1,0) is a turning point (pointing to the point in picture 1), so here I draw ee.. to the left of -1 it is an up function, then if between -1 to 0 it is a descending function.*

Constructing the Target

S2 understands the components in the target by sketching the graph f as a piecewise graph. S2 builds the components in the target by using the concept of monotony, concavity and dividing the graph f' by 3 intervals based on the shape of the graph to determine the graph sketch of $f(x)$. In the interval (-∞,0), the graph of f' is in the form of a curve (a quadratic function), the interval [0,1] and (1,∞) is a straight line (a linear function). This is in accordance with the following statement S2.

> *S2 : (starts drawing on answer paper) Here I want to draw eee.... the coordinate plane according to the graph. (after drawing the Cartesian coordinates, silently watching the problem) This seems to be a function... it's a piecewise function. This is if f(x) is (write next to the Cartesian coordinates) for what is the interval -∞ to 0, then 0 to 1, then 1 to. Mmmm ..... (looking at the problem) this is a quadratic function*

(indicating the first interval), this is a linear function (indicating the second interval), this is linear (indicating the third interval).

S2's efforts in determining the graph f using the concepts and known elements of the

S2's efforts in determining the graph f using the concepts and known elements of the problem have characterized the occurrence of the assimilation process. The result of the work of the S2 graph corresponding to the statement S2 is presented in Figure 10 as follows:

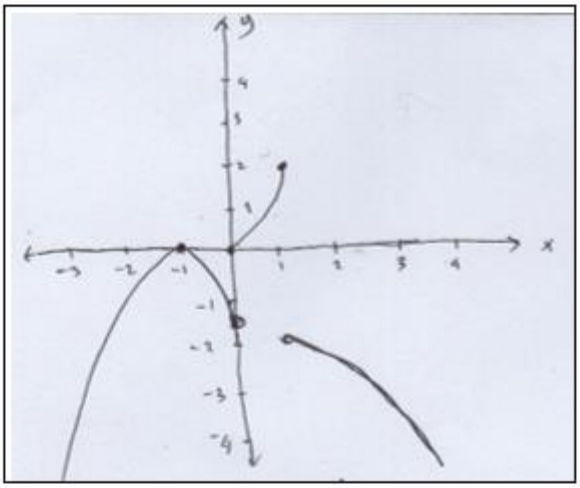

Figure 10. Representation of S2 Target Results

Determining equivalence

S2 evaluates by re-checking the work he has obtained. S2 believes that the graph of $f(x)$ is piecewise. However, when S2 was asked to re-read the questions, S2 had doubts about the results of the graph sketch of the answer. S2 improves the relationship between source and target by reading the problem again and stopping at the statement "if f is a continuous function". However, S2 did not make any changes to the answers to the $f(x)$ graph sketch that had been obtained. This is in accordance with the excerpt of the interview with S2.

P	:	*From your picture it looks almost the same, right? Piecewise pieces like that?*
S2	:	*Yes*
P	:	*Why did you think of such a form, why not a connected form?*
S2	:	*ee... because I think that for example at the intersection of (-1,0) and (0,0) the position will remain the same, because it will be the turning point*
P	:	*Have you checked from the problem?*
S2	:	*ee. if f(0) is 0 at the point. From the questions I get only f(0) is equal to 0, it means here 0 (pointing to the answer).*
P	:	*Have you read the question again?*
S2	:	*Still, (read) if f is a continuous function ... (silence) eee like that sentence*
P	:	*Why is that?*
S2	:	*(Silent looking at the problem) A continuous function must meet 3 conditions, if f(a) exists, there is a limit f(a), and the value of the same limit f is the same. (smile)*

Category III: Symbolic-Concept Translation Process

Unpacking the Source

S3 represents the elements of the problem symbolically by writing them on a worksheet. S3 is unaware of the precision of limits on intervals and whether or not to use the equal sign. This activity indicates that S3 is carrying out assimilation activities even though S3 is unaware of its incorrect statements. This can be seen from the following snippet of the interview with S3.

P : Is the information you got in the question correct?
S3 : Well that's that then
P : Is the interval right?
S3 : already

Furthermore, S3 directly interprets the components in the problem by determining the equation of the function in each interval. This can be seen from the S3 statement, namely, "I can make the function right away.". The fluency of S3 in representing the elements in the problem symbolically shows that S3 has carried out assimilation activities. The results of S3's work are shown in Figure 11 below.

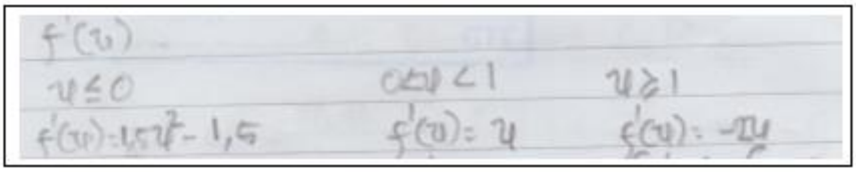

Figure 11. Results of S3 Exploration Identifying Problems

Next, S3 traces the related components in the source representation that f is a continuous function and the graph form f' is a discontinuous function. However, when the researcher asks the reason why f' is a discontinuous function, S3 cannot provide a reason (accommodation).

Preliminary Coordination

S3 assumes source and target as derived and anti-derivative relationships. S3 directly tries to find the equation f by using the concept of integral (assimilation). The symbolic representation of S3 can be seen in the work of Figure 12 below.

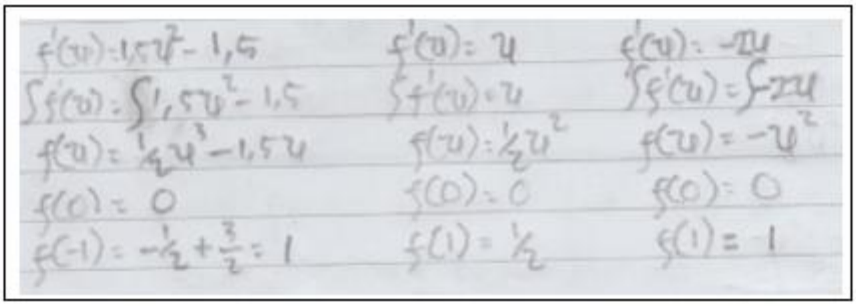

Figure 12. *Results of S3 Exploration in Determining the Graph Equation f*

Constructing the Target

S3 builds the components in the target by directly drawing a graphic sketch of the elements that have been obtained from the previous stage, namely the points on the graph and the graph equation f. S3 thinking structure when using elements that have been obtained and is known to show the occurrence of assimilation activities which are in accordance with the structure of the problem.

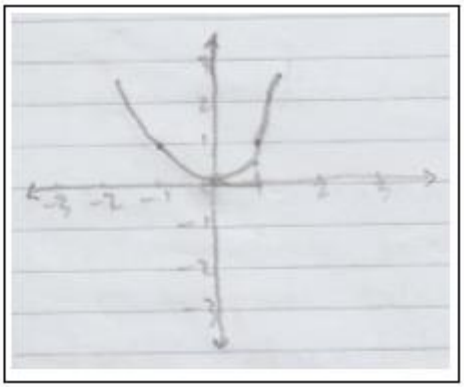

Figure 13. Figure 13 Graph Result f by S3

S3 understands the target components that have been made unsuccessful, so reflections are carried out. S3 did a reflection by looking back at the results of the graph he had drawn. This S3 activity shows the accommodation process where the thinking structure is not in accordance with the problem structure. This is in line with S3's statement, "I think the picture doesn't fit". S3 tried other ways of solving the problem until he finally remembered the concepts of monotony and hollowness. The results of the S3 graph sketch are shown in Figure 14.

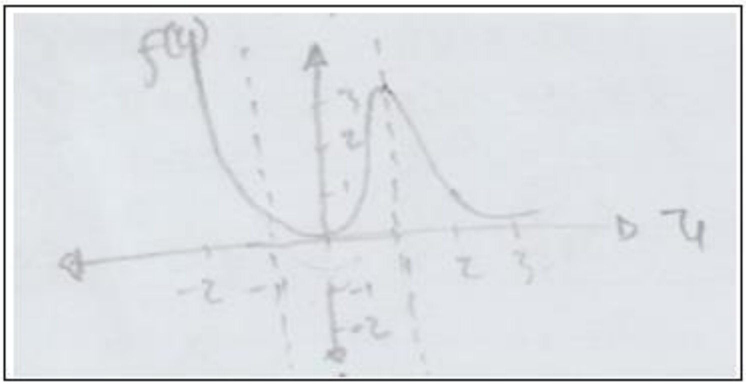

Figure 14. Results of the Representation of S3 Targets

Determining equivalence.

S3 evaluates by re-checking and improving the work he has obtained. S3's reflection shows the occurrence of accommodation activities. S3 crossed out the interval (-∞,0) on the downward concave section and added an interval (-∞,0) on the upward concave section of the hose. S3 also corrects the graph of f to be concave upwards at the interval (0,1). This can be seen from the work of S3 in Figure 15 which is marked below.

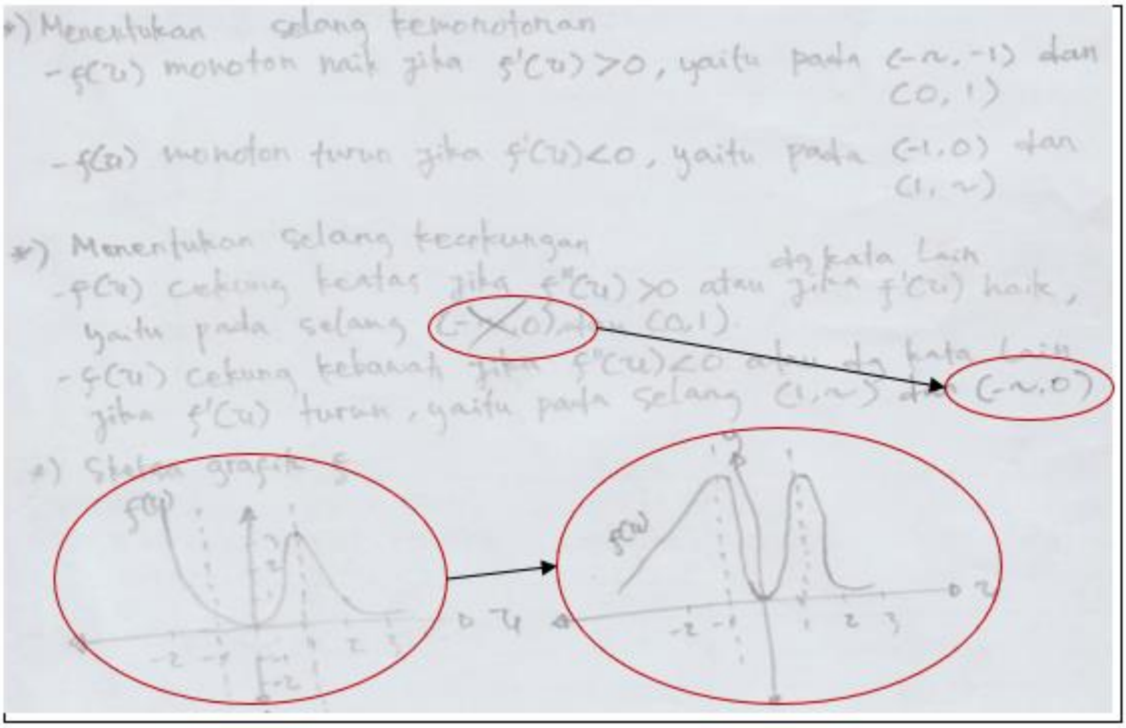

Figure 15 Results of Improvement of S3 Target Representation

Discussion

Unpacking the Source (US) Activity

In this activity, the subject identifies the components of the problem which are carried out predominantly through the assimilation process. The subject begins with the activity of identifying the components of the problem by mentioning the components in the $f'(x)$ graphic image, namely the shape of the graph, the point that passes through the graph, the interval on the graph. However, the subject paid less attention to the required source component (Bossé et.al., 2014) so it went with the wrong solution. Next, the subject interprets the components of the problem that the dominant subject performs accommodation activities. Furthermore, the subject traces the components of the problem by processing the relationships between components and relationships with the schema they have. Subjects who experience disequilibration make accommodations to track components so that equilibrium conditions are achieved. The three subject categories show the process of unpacking the source by carrying out an analytical process by describing the components of the problem (Subanji, 2011). Unpacking the source activity is illustrated in Figure 16 below.

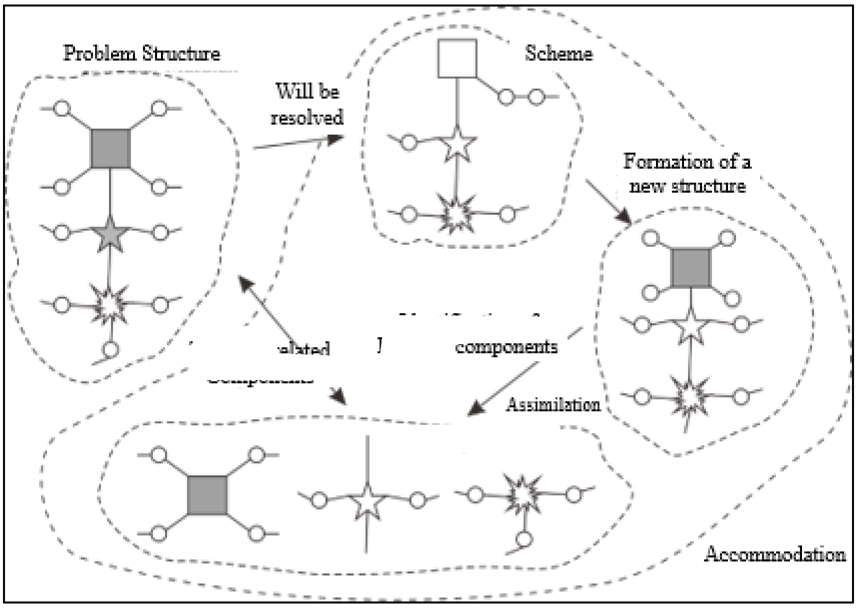

Figure 16. Unpacking the Source in Solving Problems

Preliminary Coordination (PC) Activity

Subjects in the preliminary coordination activity made assumptions about the relationship between the representation of the problem (the source) and the representation of the solution to the problem (the target). In that time, the thinking structure of the subject is in accordance with the structure of the problem so that an assimilation process occurs. Furthermore, the subject connects the components that are known to the problem (source) with components in problem solving (target). The subject makes equations of quadratic functions and linear functions by using the existing scheme (Figure 17). The activities carried out by the three categories of subjects aim to form a network of ideas in describing the relationship between mathematical representations (Bossé et.al., 2014).

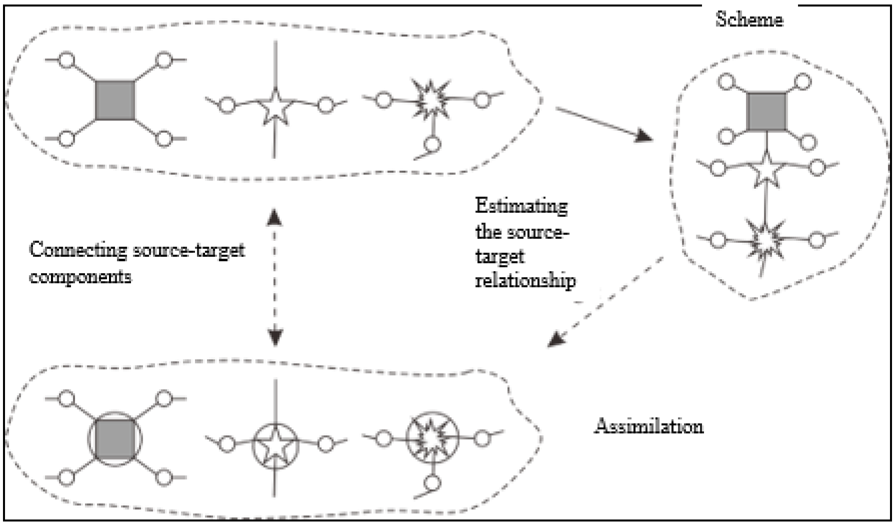

Figure 17. Preliminary Coordination in Solving Problems

Constructing the Target (CT) Activity

The subject performs the activity of constructing the target by building components on the problem solving (target) using the components in the problem. The three subjects showed activities aimed at forming the target representation based on the component network in the source representation and the target representation in the preliminary coordination (Bossé et.al., 2014). To complete the components on the graph f (target), the subject understands the components on the target by applying the continuous nature of the graph f and $f(0) = 0$ which is the component of the graph f. This shows that the schema does not match the problem structure which is an accommodation process (Figure 18).

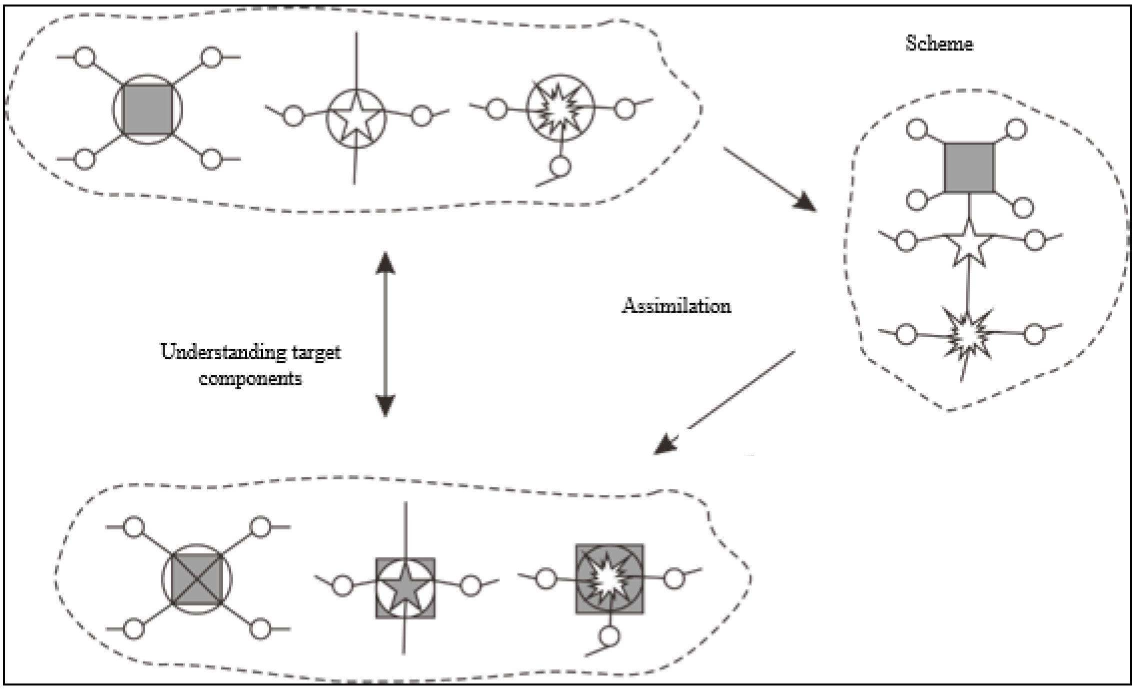

Figure 18. Constructing The Target in Solving Problems

Determining Equivalence (DE) Activity

Subject evaluates the suitability of the problem (source) with the solution that has been obtained (target). Subjects with schemas that match the problem structure have confidence in the solutions that have been obtained. This shows that the assimilation process occurs so that an equilibrium condition occurs. Meanwhile, subjects who do not have a schema match with the structure of the problem are disequilibrated so that an accommodation process occurs. Verbal translation subjects have evaluated the suitability of the source and target as the goal of the end of the translation process (Lesh dkk., 1987). The subject improves the relationship between the source and the target to achieve an equilibrium condition. The subject repeats the completion steps that have been carried out or examines the algebraic calculations carried out. In addition, there are also subjects who modify the solution steps by recalling the scheme they have to complete the completion process, such as adding the concept of limits after carrying out the integral process of functional equations (Figure 19). Subjects show that they have experienced cognitive conflict by adapting when experiencing difficulties in dealing with problems (Dewey dalam (Rodgers, 2002).

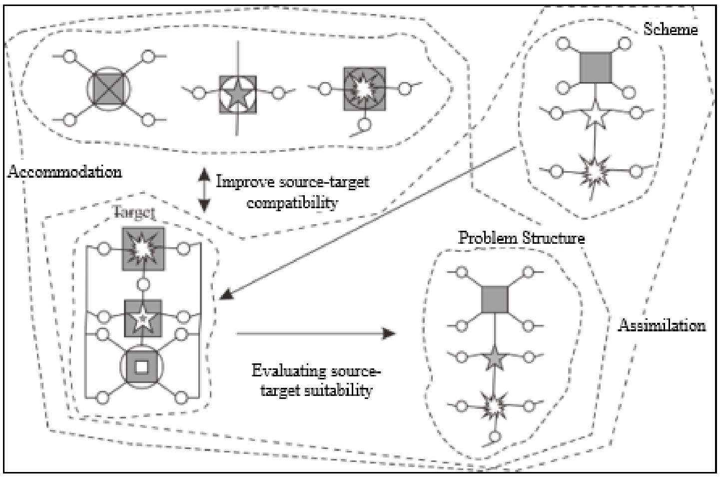

Figure 19. Determining Equivalence in Solving Problems

The translation process of representation in solving mathematical problems based on assimilation and accommodation processes can be illustrated in Figure 20 below.

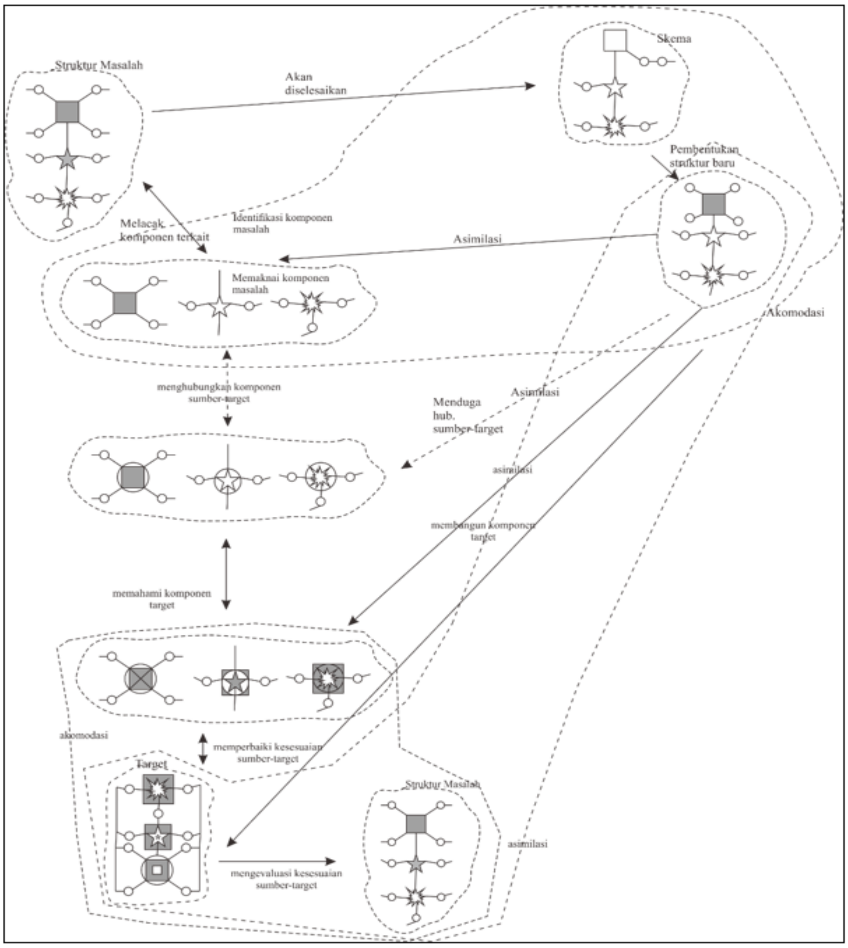

Figure 20. The Translation Process of Representation in Resolving Problems

The comparison of the three characteristics of the translation process is presented in Table 4 below.

Table 4 *Comparison of Characteristics of SA, V, and SC Mathematical Translation Processes*

Characteristics of the Translation Process		Symbolic-Algebraic Translation (SA)	Verbal Translation (V)	Symbolic-Concept Translation (SC)
US	**Identify** the components of the problem.	• The subject takes a long time to mention the source component (graphic form).	• The subject directly mentions the components of the source with verbal representations.	• The subject can directly name the source component (a graph).
	Interpret the components of the problem.	• The subject interprets the source (a graph) as an algebraic representation.	• The subject interprets the source as a piecewise function.	• The subject experiences cognitive conflicts in understanding the components of the source
	Tracking related components.	• The subject determines the algebraic form of the source component.	• The subject determines the properties of the source components.	• The subject carries out a reflection of the cognitive conflicts experienced.
PC	**Estimating** source and target relationships.	• With the scheme that the subject has, he sees the source and the target as having an antiderivative relationship.	• With the schema that the subject has, the source and the target have associated graphic properties.	• The subject uses the scheme of the properties of the graph on the source and as an antiderivative of the target component.
	Connecting source and target components.	• The subject applies the scheme by creating algebraic equations from the source component.	• The subject applies the scheme by outlining the nature of the source component.	• The subject applies the scheme by describing the properties of the source and making algebraic equations from the source.
CT	**Build** components on target	• The subject coordinates the components on the PC with a owned but incomplete scheme	• The subject coordinates the components on the PC in full.	• The subject coordinates the components on the PC but cognitive conflicts occur with the incompatibility of solutions.
	Understanding the components in the target	• The subject completes the target component by looking at the required conditions.	• The subject associates the schema with the necessary conditions of the target component.	• The subject completes the necessary conditions of the target component according to the scheme possessed.
DE	**Evaluate** the suitability of sources and targets.	• The subject performs a recheck.	• The subject performs a recheck.	• The subject performs a recheck.

Characteristics of the Translation Process	Symbolic-Algebraic Translation (SA)	Verbal Translation (V)	Symbolic-Concept Translation (SC)
Improve source and target relationship.	• The subject modifies the completion step.	• The subject recalls the schema it possessed to supplement the shortcomings of the target component.	• The subject uses other means to obtain the target component precisely.

Conclusion

Based on the framework of the accommodation assimilation process, we obtained the characteristics of the translation process of representation in each of the categories. When performing "unpacking the source," the symbolic-algebraic subject faces many questions in his mind when **identifying** the representation of the source. The symbolic-algebraic subject **interprets** the source component as a **traceable** function by determining the equation of its function. At the "preliminary coordination" activity, all subjects **estimate** that the source-target relationship is a derivative function. However, the symbolic-algebraic subject experiences when **connecting** the source and target components of the concept to which it belongs. Symbolic-algebraic and symbolic-conceptual subjects experience accommodation when the activity is "constructing targets". This can be seen from the difficulty faced by both subjects when **building** and **understanding** the components of the target representation. The activity of "determining equivalence" is characterized when the subject **evaluates** the suitability of sources and targets by double-checking the resolution of the problem. After that, the **improvement** of the source relationship carried out by the symbolic-algebraic subject is carried out by replacing the method of solving with the theory of monotony.

Other findings in this study could be the subject of future research. First, the study showed that of the 65 students who completed the translation assignment on representation, only four completed it correctly. Furthermore, subjects with symbolic-algebraic translational processes have difficulty performing “unpacking the source” and “constructing the target”, so they predominantly carry out the accommodation process. In addition, subjects with symbolic-conceptual translational processes make the transition of representations from algebra to the theory of graphic monotony when the activity is "determining equivalence". This opens up opportunities for future research on the difficulties, obstacles, or errors of algebraic symbolic subjects as well as representation transitions in the translation process of representation.

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